



## **A model of surface gravity waves on a turbulent fluid and vertically sheared flows**

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Surface gravity waves mostly govern the flow in the thin layer that connects the ocean and the atmosphere, hence they have a significant effect on momentum, energy, heat and mass exchanges between the air and water layers. The common modeling approach in large-scale wave models utilizes Bretherton and Garrett's (1968) wave action equation (WAE), which is based on potential theory of wavetrains that propagate on vertically averaged currents. This formulation performs reasonably well, but in high energetic environments there are several additional processes that need to be taken into account. In these situations currents have significant shear and when there are significant whitecaps, there is also significant upper layer turbulence. Nevertheless, there is currently no formulation for waves over shearing currents in the presence of turbulence.

Harmonic waves over shearing currents with no turbulence can be modeled from Euler equations using the WKB approximation. The zeroth order solution yields the Rayleigh equation. Its solution describes the local vertical profile of the wave flow and its dispersion relation. The next order of the approximation yields a conservation equation (see Voronovich, 1976). In order to actually use the resulting formulation, the Rayleigh equation first needs to be solved to yield the eigenvalues (frequencies/wave celerities) and eigenfunctions (vertical profiles). Then, its solution can be substituted into the conservation equation to create a WAE for wave over shearing currents. For constructing an approximated equation applicable for wave forecasting models (see Quinn et al., 2017) one can use a perturbation solution of the Rayleigh equation such as Skop (1987).

In order to use the same approach for the wave over turbulent flow problem, the present work applies WKB approximation to the Reynolds-averaged Navier-Stokes equation. The zeroth order solution is used together with appropriate boundary conditions to construct an augmented Orr-Sommerfeld type equation that extends Rayleigh's with a vertically changing eddy viscosity component. The equation is solved numerically in order to inspect the influence of various eddy viscosity profiles and vertically shearing current profiles. Preliminary results of a perturbation solution such as Skop's for the Orr-Sommerfeld equation will also be presented.

Bretherton, F.P. and Garrett, C.J.R. "Wavetrains in inhomogeneous moving media", Proc. of the Royal Society of London. Series A. Math. and Phys. Sci. 302, 1471 (1968), pp. 529-554.

Quinn, B.E. and Toledo, Y. and Shrira, V.I., "Explicit wave action conservation for water waves on vertically sheared flows", Ocean Modelling 112 (2017), pp. 33-47.

Skop, R. "Approximate Dispersion Relation for Wave-Current Interactions", Journal of Waterway, Port, Coastal, and Ocean Engineering 113, 2 (1987), pp. 187-195.

Voronovich, A.G. "Propagation of surface and internal gravity waves in geometric optic approximation", Atmo. and oceanic physics 12 (1976), pp. 850-867.