



## Conservation laws and ensemble Kalman filter algorithms

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In order to better predict the properties of nonlinear flow with numerical weather prediction (NWP) models, Arakawa (1966,1972) proposed that numerical discretization schemes preserve the most important conservation properties of the continuum system. Depending on data assimilation method, physical properties might not be preserved in analysis. In this talk, we show that data assimilation algorithms can and should incorporate some of the properties of the nonlinear flow, following the similar principle of design as NWP models.

We illustrate benefits of conservative data assimilation procedures in very simple idealized setups designed for convective scale data assimilation as well as in setups more appropriate for global data assimilation. Following the conservation principle, first conservation of mass and preservation of positivity have been shown in Janjic et al. 2014 to be important constraints for data assimilation algorithms. These two constraints have been incorporated into a new algorithm, based on the ensemble Kalman filter (EnKF) and the quadratic programming (QPEns: Quadratic Programming Ensemble filter). In Janjic et al. 2014 it was shown on a very simple example that for state estimation, the inclusion of the constrained estimation can improve the EnKF results in case of strongly non-Gaussian error distributions. The QPEns algorithm was further tested on the modified shallow water model (Würsch and Craig, 2014), which was designed to mimic the important characteristics of convective motion. It was shown that the mass conservation- and positivity-constrained rain significantly suppresses noise seen in localized EnKF results. In addition, the root mean square error (RMSE) is reduced for all fields and total mass of the rain is better simulated.

Further, using perfect model experiments with mass, total energy and momentum conserving 2D shallow water model that also conserves enstrophy for non-divergent flow, we illustrate the effects that data assimilation algorithm has on these quantities (Zeng and Janjic, 2016). During assimilation, the total mass remained consistent with that of the nature run and the total energy of the analysis mean converged towards the nature run value. However, enstrophy, divergence, as well as energy spectra were strongly affected by localization radius, thinning interval, and inflation and were dependent on the variable that is observed. For all localization radiuses for which RMSE was reasonable, the enstrophy has increased and converged to the value significantly higher than globally integrated enstrophy of nature run. Finally QPEns algorithm was extended in order to preserve nonlinear equality constraints such as conservation of energy and enstrophy (Zeng et al. 2017). Although all experiments with constraints exhibited comparable RMSE, the kinetic energy and enstrophy spectra in experiments with the enstrophy constraint (on the globally integrated enstrophy) were considerably closer to the true spectra, in particular at the smallest resolvable scales. Therefore, imposing conservation of enstrophy within the data assimilation algorithm effectively avoids the spurious energy cascade of rotational part and thereby successfully suppresses the noise. The 14-day deterministic and ensemble free forecast, starting from the initial condition enforced by both total energy and enstrophy constraints, produced the best prediction.