



Stochastic geophysics, multifractal vector fields and examples of Lie algebra of their generators

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For decades most of the developments of theoretical geophysics had been based on deterministic processes, the need of statistical analysis has been progressively accepted and more recently of stochastic processes. In particular, multifractals or multiplicative chaos have become iconic of nonlinear processes with multiple scaling singularities. However multifractals have been for too long restricted to be scalar valued, i.e. to have one-dimensional codomains. This has prevented to deal with the key question of complex component interactions of vector fields and their non trivial symmetries.

This limitation was recently overcome by considering the Lie algebra of stochastic generators of cascade processes with arbitrarily large codomains, e.g., flows of vector fields over large dimensional manifolds. In this communication, we first focus on the case of stable Levy generators on Clifford algebra, then point out some generalisation. We introduce Clifford algebra with the help of symmetries as simple as orthogonal rotations and mirror symmetries that square respectively to minus and plus unity. This enables to underline the role of spherical and hyperbolic geometries that are respectively associated to square roots of minus and plus unity. Hyperbolic subspaces require extremely asymmetric Levy noises to obtain finite statistics, whereas there is no constraint on spherical eigenspaces. We also show that they generate across scales flows of their respective types. We finally consider generalisation to other Lie algebra with the help of their Killing form.

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