

Commutative Jordan algebras and binary operations: a framework on environment data

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Models with orthogonal block structure having variance-covariance matrices

$$V = \left\{ \sum_{j=1}^m \gamma_j Q_j \right\},$$

where the Q_j are known pairwise orthogonal orthogonal projection matrices and $\gamma_1, \dots, \gamma_m$ are unknown non negative constants, continue to play an important part in the theory of randomized block designs.

We now study an important class of these models, these with commutative orthogonal block structure, in which T , the orthogonal projection matrix on the range space spanned by the mean vector commutes with Q_1, \dots, Q_m .

We use commutative Jordan algebras of symmetric matrices in order to study the algebraic structure of these models and to obtain unbiased estimators.

This framework, along with the construction of models based on the algebraic structure, through the use of the binary operations that are associated to commutative Jordan algebras, will provide significant advantages upon the treatment of large quantities of data, along with the computational methods.

An application to environment data will be provided, thus showing the advantages of the use of these structures on environmental data.

Keywords: COBS, Inference, Commutative Jordan algebras.

Acknowledgements

This work was partially supported by UID/MAT/00297/2013 and by European Investment Funds by FEDER/COMPETE/POCI Operational Competitiveness and Internationalization Program, under POCentro-PT2020-FEDER project Centro-01-0145-FEDER-024253

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