



On canonical vortex dynamics for ball lightning

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Our boson model treats ball lightning as turbulent plasmoid threaded by coherent structures of string-like vortex filaments. Our real matrix format for split-quaternion space has kinematic solutions for spherical eigenstates characterized by chiral flow patterns [1]. Torsion adds swirling motion in eigenstates found canonical by criteria for Hamiltonian dynamics and symplectic matrices [2]. Split-quaternions differ from the regular ones only by sign changes in their algebra and calculus rules. Partial derivatives of matrix S in Eq. 1 define four elementary matrices for split-quaternion algebra. Our real matrix format reduces conjugation to transposition S^T , confirming the symplectic nature of S through:

$$S \equiv \{w, -x, y, z\}, \{x, w, z, -y\}, \{y, z, w, -x\}, \{z, -y, x, w\}, \quad S.J.S^T = J \quad (1)$$

with $u \equiv \{1, 0\}, \{0, 1\}$ as unit matrix in Jordan matrix $J \equiv \{\{0, u\}, \{-u, 0\}\}$. Axes $\{w, x\}$ and $\{y, z\}$ define complex sub-planes. Matrix S also resolves split-quaternion space into a collinear triplet $\{w, x\}, \{w, y\}, \{w, z\}$ of one complex and two split-complex planes respectively. Sub-domains of S-space admit calculus for standard functions meeting six local and six global regularity conditions validated by inductive proofs [3]. Eigenstates of S reveal chiral degrees of freedom for swirling solutions independent of co-ordinate w [1]. Within ideal fluids vortex lines must close into loops with invariant circulation and flux as proved by Helmholtz and Kelvin [4]. For planar vortex motion in such fluids Kirchhoff derived Hamiltonian forms $dx/dt = -\partial W/\partial y$, $dy/dt = \partial W/\partial x$ [4, 5], which we extend into 4D (split-)quaternion space through:

$$W \equiv \log(w^2+x^2+y^2+z^2) \Rightarrow dw/dt = -\partial W/\partial x, dx/dt = \partial W/\partial z; dy/dt = -\partial W/\partial z, dz/dt = \partial W/\partial y \quad (2)$$

Beyond hydrodynamics Lamb's treatise explicitly states that vortex filaments correspond to electric currents, and fluid velocity corresponds to magnetic force in formulas of electromagnetism. Thus our model finds the natural ball lightning with its stunning energetics potential deeply rooted in Hamiltonian dynamics.

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