

## Flow-induced Coordinates and Reconstruction Techniques for Transient Advection-Diffusion Equations with Multiple Scales

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Simulation over a long time scale in climate sciences as done, e.g., in paleo climate simulations require coarse grids due to computational constraints. Coarse grids, however, leave important smaller scales unresolved. Thus small scale processes that significantly influence the resolved scales can either be neglected (which is not desired) or their influence has to be taken care of by different means. Such processes include (slowly) moving land-sea interfaces or ice shields as well as flow over urbanic areas. State-of-the-art dynamical cores represent the influence of subscale processes typically via subscale parametrizations and often employ heuristic coupling of scales.

We aim to improve the mathematical consistency of the upscaling process that transfers information from the subgrid to the coarse prognostic scale (and vice-versa). We investigate new bottom-up techniques for advection dominated problems arising in climate simulations [Lauritzen et al.,2011]. Our tools are based on ideas for multiscale finite element methods for elliptic problems that play a role, in oil reservoir modeling and porous media in general [Efendiev et al.,2009; Graham et al.,2012]. Modifying these ideas is necessary in order to account for the transient and advection dominated character that is typical for flows encountered in climate models.

We present a new Galerkin based idea to account for the typical difficulties in climate simulations. Our modified idea employs a change of coordinates based on a coarse grid characteristic transform induced by the advection term in order to account for appropriate subgrid boundary conditions for the multiscale basis functions. The boundary conditions are essential for such approaches. We discuss extensions and drawbacks of this approach and propose a solution based on a reconstruction of small scale features in basis functions in a semi-Lagrangian framework. We also present results from sample runs for a simple advection-diffusion equation with rapidly varying coefficients on several scales.