



Learning an optimized dictionary for gravity field modelling

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In the last decade, the idea of a matching pursuit has been used for the iterative solution of ill-posed inverse problems in the geosciences, for instance the downward continuation, the inverse gravimetric problem and normal mode tomography. The main advantages of the developed algorithms, the (Regularized) Functional Matching Pursuit ((R)FMP) and the (Regularized) Orthogonal Functional Matching Pursuits ((R)OFMP), are that we do not need to solve a system of linear equations or invert a matrix anymore. Furthermore, we can make use of the local and global structures of different types of trial functions, for instance radial basis functions (RBFs) and spherical harmonics. Hence, we obtain a multiresolution of a problem at hand. The idea is to use an overcomplete but in practice naturally finite dictionary which includes local and global trial functions to iteratively build a best basis and compute a (e.g. gravity) model in this best basis.

Experiments show that the choice of this dictionary is crucial for the obtained approximation. Thus, the question at hand is, whether there is an optimal strategy for choosing a finite dictionary from the infinite set of trial functions. We present an approach which enables us to find an optimized finite subset of certain RBFs (Abel-Poisson kernels) and spherical harmonics. This 'learned' dictionary has the potential to essentially improve the performance of the matching pursuits. Moreover, dictionaries can be learned specifically for certain application scenarios such as gravity field modelling.

For the learning strategy, we model the objective function with respect to the kernels as a non-linear optimization problem in every iteration step of the matching pursuit. This is solved using the Ipopt software package with an HSL subroutine and yields a new candidate for being an element of an optimized dictionary. We decide whether it is inserted into the optimized dictionary by considering its effect on the decrease of the relative error of a given problem.

In our presentation, we explain the idea of our learning algorithm and demonstrate numerical examples with respect to the EGM2008.