A construction manual for Slepian functions to regularize regional ill-posed inverse problems

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Slepian functions are orthogonal function systems that live on subdomains (for example, geographical regions on the Earth's surface or bandlimited portions of the entire spectrum). They have been firmly established as a useful tool for the synthesis and analysis of localized (concentrated or confined) signals, and for the modeling and inversion of noise-contaminated data that are only regionally available or only of regional interest. In particular, they turned out to be useful for the (regional) modelling and inversion of potential fields.

Though Slepian functions have been constructed for various domains and direct or inverse problems, they all have common features and building blocks. This can be shown be representing the Slepian approach in an abstracted setting with a linear and compact operator. This point of view allows us to describe a construction manual for building Slepian functions for any specific inverse problem with regional data. A necessary ingredient is the knowledge of the singular-value decomposition (svd) of the operator which represents the forward problem with global data (such decompositions are known for common inverse problems associated to gravity and magnetic data). We explain how Slepian functions can be constructed depending on this svd and an arbitrarily chosen region. The obtained Slepian basis can be used to easily derive an svd for the regional inverse problem. This new svd, itself, enables us to apply further advanced regularization techniques such as wavelet-based methods.

We demonstrate the methodology on a simple, synthetic example on the 1-sphere.