



## Minimal-Discrepancy Point Grids on Balls for Numerical Methods in Earth Sciences

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Quantifying, how well a set of points on a given domain is distributed, is a widely discussed topic. In Geosciences, the domain of a ball is of particular interest, because of its resemblance with the Earth. The problem has already been considered on the surface of the ball and a solution in the form of a generalized discrepancy has been provided by Cui and Freeden [1]. This work aims to extend this idea of the generalized discrepancy to the ball of arbitrary dimensions and to study its properties. Extending this idea to the domain of a solid ball not only enables us to have favorable configurations on the ball but also convenient and applicable integral approximations of functions on the ball. Consequently, the concept of generalized discrepancy helps us achieving two aspects at the same time. The low discrepancy patterns on the ball, for instance, can be used as the centres of the radial basis functions as they are required for the regularization methods like the RFMP algorithm and the ROFMP algorithm, developed by the Geomathematics Group at the University of Siegen for ill-posed inverse problems with particular focus on the sphere and the ball as domains of the unknown functions. Moreover, well distributed points on the ball are also useful for some other numerical methods. An example is represented by certain wavelet methods, where one needs to have an appropriate quadrature rule.

## References

- [1] J. Cui, W. Freeden: *Equidistribution on the sphere*. SIAM Journal on Scientific Computing, 18:595-609, 1997.
- [2] A. Ishtiaq, V. Michel: *Pseudodifferential operators, cubature and equidistribution on the 3D-ball – an approach based on orthonormal basis systems*. Numerical Functional Analysis and Optimization, 38:891-910, 2017.