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An unexpected uniqueness result for tomography-assisted potential-field inversion

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Inversion of potential field data is a central technique of remote sensing in geophysics, but also in physics, neuroscience, and medical imaging. Uniqueness theorems for potential-field inversion are scarce and applied studies try to improve potential-field inversion by constraining the solution through independent information about the source location. Yet, so far no mathematical theorem guarantees that such source localization ameliorates the inversion in terms of uniqueness of the achieved assignment. Here, standard potential field theory is used to prove a uniqueness theorem which completely characterizes the mathematical background of source-localized inversion. It guarantees for an astonishingly large class of source localizations that it is possible by potential field measurements on a surface to differentiate between signals from a finite number of source regions. The main result is the following theorem:
Let $\Omega \subset \mathbb{R}^3$ be open, simply connected, and $\partial\Omega$ a smooth compact manifold. Assume that $P_1, \ldots, P_N \subset \Omega$ are pairwise disjoint compact sets such that

$$\mathbb{R}^3 \backslash P_k \text{ and } \mathbb{R}^3 \backslash \bigcup_{i=1}^{k} P_i$$

are simply connected for all $k = 1, \ldots, N$. If the sources of the zero-gauged potential Φ have compact support on

$$\bigcup_{k=1}^{N} P_k,$$

then

$$\frac{\partial \Phi}{\partial n} \text{ on } \partial\Omega$$

uniquely determines zero-gauged potentials $\Phi_1, \ldots, \Phi_N$, such that Φ_i is harmonic on

$$\mathbb{R}^3 \backslash \bigcup_{k \neq i} P_k,$$

which implies that it has no sources outside P_i, and

$$\frac{\partial \Phi}{\partial n} = \sum_{i=1}^{N} \frac{\partial \Phi_i}{\partial n} \text{ on } \partial\Omega.$$

Non-uniqueness of potential field inversion only prevents that the source distribution within the individual regions can be uniquely recovered. The new theorem is a substantial generalization of a theorem of Gauss about the separation of the sources of the geomagnetic field. In our research area it provides the theoretical background for a new paradigm of rock- and paleomagnetic measurements based on studying individual sub-micron particle magnetizations.