Two phenomenological constants explain similarity laws in stably stratified turbulence

Gabriel Katul (1) and Dan Li (2)

(1) Duke University, Nicholas School of the Environment, Durham, United States (gaby@duke.edu), (2) Department of Earth and Environment, Boston University, Boston, Massachusetts, USA

In stably stratified turbulent flows, the mixing efficiency associated with eddy diffusivity for heat, or equivalently the turbulent Prandtl number ($Pr_t$), is fraught with complex dynamics originating from the scale-wise interplay between shear generation of turbulence and its dissipation by density gradients. A large corpus of data and numerical simulations agree on a near-universal relation between $Pr_t$ and the Richardson number ($R_i$), which encodes the relative importance of buoyancy dissipation to mechanical production of turbulent kinetic energy. The $Pr_t - R_i$ relation is shown to be derivable solely from the co-spectral budgets for momentum and heat fluxes if a scale-wise Rotta-like return to isotropy closure for the pressure-strain effects and Kolmogorov’s theory for turbulent cascade are invoked. The ratio of the Kolmogorov to the Kolmogorov-Obukhov-Corrsin phenomenological constants, and a constant associated with isotropization of the production whose value has been predicted from Rapid Distortion Theory ($= 3/5$), explain all the macroscopic non-linearities. Comparisons with a conventional Mellor–Yamada (MY) model for stably stratified atmospheric flows as well as the Energy- and Flux-Budget (EFB) approach is featured with a lens on the cut-off Richardson number dilemma.