

## A wave equation with friction factor and momentum correction factor of roll waves on shallow water

Muneyuki Arai

Meijo University, Faculty of Science and Technology, Civil Engineering, Nagoya, Japan (arai@ccmfs.meijo-u.ac.jp)

One factor of generation of intermittent debris flow surges in the mountain basin is based on flow instability. In this study, it was derived a wave equation including a friction factor  $f'$  and a momentum correction factor  $\beta$  as shallow water flow wave equations.

The non-dimensional basic equations are as follows, using velocity potential  $\phi'$ .

$$\partial^2 \phi' / \partial x'^2 + \partial^2 \phi' / \partial y'^2 = 0 \quad (1), \quad \partial \phi' / \partial y' = 0, \quad (y' = -1) \quad (2)$$

$$\partial \phi' / \partial y' - \partial \eta' / \partial t' - (\partial \phi' / \partial x') (\partial \phi' / \partial \xi') = 0, \quad (y' = 0) \quad (3)$$

$$\partial \phi' / \partial t' + \frac{1}{2} (2\beta + 1) (\partial \phi' / \partial x')^2 - c_0'^2 \tan \theta x' c_0'^2 (1 + \eta') + (f' / 2) u_0' c_0' \phi' + (\beta - 1) u_0' c_0' \int (\partial \phi' / \partial x') (\partial \eta' / \partial x') dx' = 0 \quad (4)$$

Equation (1) is Laplace equation by the incompressible and non-rotational condition of fluid, equation (2) a condition at the flow bottom, equation (3) the conservation condition of the displacement of flow surface and the potential, and equation (4) the fluctuation condition of flow surface. Where,  $u_0' = u_0 / c_0$ ,  $c_0' = c_0 / v_{p0}$ ,  $c_0 = \sqrt{g h_0 \cos \theta}$ ,  $u_0$  : mean velocity,  $h_0$  : mean depth,  $\theta$  : channel slope,  $g$  : acceleration due to gravity, and  $v_{p0}$  is a velocity parameter by G-M transformation,  $\xi = \epsilon^{\frac{1}{2}} (x - v_{p0} t)$ ,  $\tau = \epsilon^{\frac{3}{2}} t$ . These non-dimensional variables are defined as follows.  $x' = x / h_0$ ,  $y' = y / h_0$ ,  $\phi' = \phi / h_0$ ,  $t' = (v_{p0} / h_0) t$ ,  $\eta' = \eta / h_0$ .

Using by perturbative method, the following wave equation on fluctuation  $\eta$  from mean depth  $h_0$  was obtained. Here,  $\eta^{(1)}$  is written as  $\eta'$ .

$$\partial \eta' / \partial \tau' + \frac{1}{2} \left\{ (2\beta + 1) c_0'^2 + (\beta - 1) u_0' c_0' \right\} \eta' (\partial \eta' / \partial \xi') - \frac{1}{4} \left( 1 / c_0'^2 - 1 / 2 \right) f' u_0' c_0' \left( \partial^2 \eta' / \partial \xi'^2 \right) + \frac{1}{2} \left\{ \left( 2 + c_0'^4 / \left( 2 c_0'^2 \right) - 3 / 2 \right) \right\} \left( \partial^3 \eta' / \partial \xi'^3 \right) = 0 \quad (5).$$

The second term, the nonlinear term of the above equation is related to the momentum correction factor, and the third term, the dissipation term is related to the friction factor and the momentum correction factor. Neither of them is related to the fourth term, the dispersion term.

These facts are understood to mean that flow models are involved in the formation and characteristics of intermittent surge waveforms and are not very involved in their propagation.