

Nonlocal Model for Deep Water Waves of a Potential Flow

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The present paper offers a fully consistent scheme of reducing the spatial dimension of nonlinear deep-water wave theory by introducing the concept of fractional derivatives along the free surface. Starting from the 2D Euler equations for an incompressible potential flow, a (1+1)-dimension model (space and time) describing deep water surface waves is derived. Similar to the shallow-water-case, the z -dependence of the dependent variables is integrated out and a set of two equations for the surface velocity $u_s(x, t)$ and the surface elevation $\eta(x, t)$ remains. The model is nonlocal and can be formulated in conservative form, describing waves over an infinitely deep layer. It is based on a systematic expansion of the governing equations with respect to η and can be extended to higher orders as well as to three spatial dimensions, resulting in a 2+1-dimensional set of three PDEs for $\eta(x, y, t)$, $u_s^x(x, y, t)$ and $u_s^y(x, y, t)$.

Finally, numerical solutions are presented for different initial conditions. Coherent wave trains are shown to be unstable due to side-band instability (Benjamin-Feir instability) and localized solutions are obtained.