



An efficient three-dimensional semi-implicit finite volume scheme for the solution of coupled free-surface and variably saturated sub-surface flow

Federica Gugole (1,2), Michael Dumbser (3), and Guus Stelling (4)

(1) Meteorological Institute and Center for Earth System Research and Sustainability, University of Hamburg, Hamburg, Germany (federica.gugole@uni-hamburg.de), (2) Department of Mathematics, University of Trento, Trento, Italy, (3) Laboratory of Applied Mathematics, University of Trento, Trento, Italy, (4) Stelling hydraulics B.V., Hoorn, Netherlands

A novel mass conservative semi-implicit finite volume scheme for the simulation of coupled free-surface and variably saturated non-hydrostatic sub-surface flows is presented. The free surface flow domain is assumed to have a small vertical scale as compared with the horizontal extents. Therefore the free-surface dynamics is described by the 2D shallow water equations, which can be obtained from the 3D Navier-Stokes equations after depth averaging and assumption of a hydrostatic pressure. No simplifying assumptions are made for the subsurface flow, which in our model is governed by the fully three-dimensional non-hydrostatic Richards equation, solved in mixed form. The coupling is achieved in a consistent and fully mass conservative way by writing both the 3D non-hydrostatic Richards equation and the 2D depth-integrated Navier-Stokes system in a monolithically coupled form, which has the three-dimensional distribution of the pressure head for both subsurface and free surface flow as the only unknown. After a conservative finite volume discretization, the resulting algebraic system for the pressure is well-posed, mildly non linear and with a linear part that is symmetric and at least positive semi-definite. This mildly nonlinear system is efficiently solved by the iterative nested Newton-type method of Casulli and Zanolli that yields the pressure head of the free-surface and the groundwater flow simultaneously. Once the pressure head is known, the free surface flow velocity and the subsurface moisture content and hydraulic conductivity can be directly computed. Restrictions on the time step size are given only by the horizontal viscosity and by the discretization chosen for the advective terms of the free surface flow, and not by the bottom friction or by the celerity of the free surface waves. In the horizontal $x - y$ plane, the domain has been covered with an unstructured orthogonal grid, possibly including subgrid details, while in the vertical z direction a simple discretization with parallel z -layers is applied. The resulting algorithm is relatively simple to implement, accurate and computationally efficient. Mass is exactly conserved also in presence of wetting and drying dynamics in the free surface flow and during the transition from the free-surface to the sub-surface and vice versa. A few examples show the accuracy and effectiveness of the proposed algorithm, and finally its performance is analyzed when implemented in parallel using the MPI standard on distributed memory machines.