Ensemble averaging and the curse of dimensionality

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When comparing weather or climate models to observations it is often observed that the mean over many models has smaller errors than most or all of the individual models. We will show that a general consequence of the non-intuitive geometric properties of high dimensional spaces is that the ensemble mean often outperforms the individual ensemble members. This also explains why the ensemble mean often has an error that is 30% smaller than the median error of the individual ensemble members. The only assumption that needs to be made is that the observations and the models are independently drawn from the same distribution.

An important and relevant property of high dimensional spaces is that independent random vectors are almost always orthogonal. Furthermore, while the length of random vectors are large and almost equal, the ensemble mean is special as it is located near the otherwise vacant center. The theory is first explained by an analysis of Gaussian and uniformly distributed vectors in high dimensional spaces. A sub-set of 17 models from the CMIP5 multi-model ensemble is then used demonstrate the validity and robustness of the theory in realistic settings.