



Direct integration method for surface waves on depth dependent flows

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We analyze surface waves interacting with a current whose magnitude and direction may vary with water depth. A direct integration method (DIM) is introduced in order to seek the exact solutions of dispersion relation of waves atop a depth dependent flow using an iterative approach. Detailed analysis of the method is presented, and comparisons with existing approximations are made for several typical natural shear currents, e.g. exponential shear current and wind-induced shear current.

The DIM also has several advantages over analytical approximations that are widely used in current measurements and ocean current models, e.g. the approximate dispersion introduced in Skop (1987) and extended in Kirby and Chen (1989). Not only does it have well controlled errors and is comparable in terms of computational cost, it is also capable of obtaining arbitrary solutions. Moreover, it readily provides rich information of the fluid field, e.g. the velocity and pressure field.

Compared to the piecewise linear approximation presented in Smeltzer and Ellingsen (2017), we argue that the DIM is superior for all practical purposes – it is faster and there is no need to selecting the correct physical solutions among a number of spurious numerical solutions. The DIM also readily calculates the full flow field with little extra efforts and comes with a built-in error estimate. Compared to the shooting method due to Dong and Kirby (2012), the DIM solves a linear differential equation, making it quicker and more easily parallelizable.