



## Stochastic stability of the Lorenz [U+0092] 63 model under anthropogenic type forcing

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### 1 Abstract

The physical behaviour of turbulent systems such the atmosphere are usually modeled by flows exhibiting a sensitive dependence on the initial conditions. The behaviour of the trajectories of the system in the phase space for large times is usually numerically very hard to compute. A way to overcome this problem is to select a few of these relevant observables under the hypothesis that the statistical properties of the smaller system defined by the evolution of such quantities can capture the features of the statistical behaviour of the original system. This turns out to be the case when considering *classical Lorenz model*, a.k.a. *Lorenz'63 model* in the physics literature.

A possible way to study this problem is to add a weak perturbing term to the phase vector field generating the atmospheric flow which model the atmospheric circulation: the so called *anthropogenic forcing*. A more realistic model for the anthropogenic forcing should take into account random perturbations of the phase vector field. Anyway it seems unlikely that the resulting process can be a diffusion, since in this case the driving process fluctuates faster than what it is assumed to do in principle a perturbing term of the type just described.

For any realization of the noise  $\eta \in [-\varepsilon, \varepsilon]$ , we consider a flow generated by the phase vector field  $\phi_\eta$  belonging to a sufficiently small neighborhood of the classical Lorenz one in the  $C^1$  topology. For  $\varepsilon$  small enough, the realizations of the perturbed phase vector field  $\phi_\eta$  can be chosen such that there exists an open neighborhood of the unperturbed attractor in  $\mathbb{R}^3$ , independent of the noise parameter  $\eta$ , containing the attractor of any realization of  $\phi_\eta$  and, moreover, such that a given Poincaré section  $\mathcal{M}$  for the unperturbed flow is also transversal to any realization of the perturbed one. Thus, given  $\mathcal{M}$ , the random process describing the perturbation is constructed selecting at random, in an independent way, the value of  $\phi_\eta$  at the crossing of  $\mathcal{M}$  by the phase trajectory.

When  $\mathcal{M}$  is chosen in such a way that the corresponding Poincaré map is continuous we show that the unperturbed physical measure is stochastically stable.

Anyway, in general, we can prove that the invariant measure of the random process defined by our perturbation procedure weakly converges to the unperturbed measure of the suspension flow associated to the Poincaré map defined on  $\mathcal{M}$  as the intensity of the perturbations tends to zero, which gives back enough information on the robustness of the statistical features of the unperturbed flow.