



## Differential geometry of ice flow

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Flowlines on ice sheets and glaciers form patterns exhibiting complex spatiodynamics. A mathematical study of their differential geometry enriches our geomorphological understanding of ice routing. To quantify the geometry, we develop a theory of planimetric flowline “convergence” and “curvature” and explore the properties of these scalar fields. We discover that flowline convergence equals the negative mathematical divergence (i.e.  $\text{div}$ ) of the unit vector field of ice-flow direction, while curvature is the curl of this vector field. Analysis of these results yields integral expressions showing that ice flow within individual catchments of an ice sheet or cap can converge despite its overall (divergent) spreading, because the ice divides between them are loci of strong planimetric divergence. This is the basis of the “symmetry breaking”—transition from approximately radial flow to flow with substantial azimuthal velocity component—behind the tributarisation behaviour during the genesis of ice-stream networks. Using the results, we unravel also the topological control behind the balance-velocity fields of ice masses, finding that planimetric convergence participates in the mass conservation equation to amplify downstream ice flux via a positive feedback. This mechanism explains why balance-velocity models predict ice-stream-like tracts of fast flow along valleys of the surface topography. Our theory provides a roadmap for understanding the tower-shaped plot of flow speed versus convergence that has been found for the Antarctic Ice Sheet. An interesting future direction is to cast the Stokes Equations in curvilinear terms using planimetric convergence and curvature, in order to study the time-dependent dynamics of these fields and formulate patterning theories of ice-stream network evolution.