



Linear response theory and the Kubo fluctuation dissipation relation applied to geoengineering

Tamas Bodai (1), Valerio Lucarini (1), and Frank Lunkeit (2)

(1) University of Reading, Mathematics and Statistics, Reading, United Kingdom (t.bodai@reading.ac.uk), (2) CEN, Meteorological Institute, University of Hamburg, Hamburg, Germany

We investigate in an intermediate-complexity climate model (I) the applicability of linear response theory to a geoengineering problem and (II) the success of the considered geoengineering method. The geoengineering method is stated here in terms of a special optimal control problem, which leads mathematically to an *inverse* problem: Global climate change with respect to an appropriate ensemble average of the surface air temperature $\langle [T] \rangle$ due to a *given* rise in carbon dioxide concentration $[\text{CO}_2]$ in a time period is attempted to be *anceled* out or modulated by an appropriately chosen modulation of the solar forcing in that period. The latter, considering an infinite time period, is predicted by linear response theory in frequency-domain as: $f_s(\omega) = (\Delta \langle [T] \rangle(\omega) - \chi_g(\omega) f_g(\omega)) / \chi_s(\omega)$, where the χ 's are linear susceptibilities.

Regarding (I), we find that the model's response with respect to (wrt.) $[T]$ to a doubling of $[\text{CO}_2]$ is not so linear, while its response to solar forcing that is supposed to cancel global warming based on the above formula is linear to a good approximation. As a result of the nonlinearity, our linear prediction fails to represent precisely the solar forcing required for total cancellation, and so, when that predicted solar forcing is applied in combination with the given greenhouse forcing, there is a nonzero positive residual total response of the model, whose asymptotic value is roughly 10% of that with $[\text{CO}_2]$ -doubling alone.

Regarding (II), we *diagnose* in this geoengineering scenario the response wrt. zonal or regional averages of T too. We predict using linear response theory a typically nonzero total response with a nontrivial spatial pattern. However, because of a strongly nonlinear ice-albedo feedback, using a *finite* magnitude forcing for system identification the *susceptibilities are estimated very inaccurately* (I) in the high-latitude regions. As a result of this, the linear prediction under the combined forcing scenario is very inaccurate; and so the predicted spatial pattern is also inaccurate. The magnitude of the true total response is in fact much smaller than the erroneous linear prediction, and some cancellation is even achieved even for regional averages (II).

Even-order nonlinear terms can be simply *eliminated* from the response considered in order to more accurately estimate the susceptibilities. In fact doing this *improves significantly* on those estimates. In turn, on the one hand, this *cuts the unwanted residual* total global warming under geoengineering *five-fold*, and, on the other hand, even the spatial patterns are predicted rather well (I). However, the latter pattern implies that significant residual total response remain in typically high-latitude regions, which *questions* the practicality of this type of geoengineering (II).

Instead of via applying external forcing to the climate model, with more practical relevance, we also attempt to determine the susceptibilities and the Green's functions via the Kubo fluctuation dissipation relation from 'free' fluctuations of the unforced model. Furthermore, a parametric inference of the coarsegrained stochastic system is also pursued hoping to make greater use of the available data.