



Liouville solutions for low complexity systems: characterization of PDF evolutions

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The prediction of weather and climate is one of the most challenging problems faced nowadays by the scientific community, not only for its incalculable value for macroevolution planning but also for civil protection and management of a myriad of socioeconomic assets. One of the main causes why numerical weather forecasts made with dynamical models are uncertain is the lack of knowledge about the state of the atmosphere with infinite precision. The appropriate theoretical context to model systems with uncertain state is the probability theory, and the state of the system is described by a probability density function (PDF), which can be depicted in the phase space of the system, whose dimensions are the state variables. In this context, the Liouville equation (LE) describes the evolution of PDFs of dynamical systems. For the atmosphere, the lack of realistic analytical solutions for the Navier-Stokes equations and the very high dimensionality of the system hampers the solution of the LE. In practice, only an ensemble of discrete samples of the PDF is considered operationally, and a predicted PDF is –sometimes implicitly– inferred from the ensemble of samples of deterministically predicted states. In this work, we apply the LE to low complexity models to learn about the evolution of PDF in increasingly complex systems and identify aspects of its evolution that provide some guidance on the optimum generation strategy in real-world operational atmospheric ensemble prediction systems. The solution of the LE for some of these low complexity models is presented, such as a specific solution of the barotropic vorticity model. Results show that even for low complexity systems, the predicted PDF can display interesting topological characteristics that challenge some popular and standard interpretations of current ensemble prediction systems.