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Discrete Wave Equation Upscaling

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Waves are affected by sub-wavelength structures not in an explicit but in an effective way. Here we present a novel homogenization technique, referred to as „Discrete Wave Equation Upscaling“ (DWEU), that operates on the discrete form of the wave equation. It provides the effective medium properties directly on a coarser grid that may be used for more efficient numerical wave propagation.

Homogenization is the mathematical procedure to transform an initial detailed medium with sub-wavelength structures into an effective medium that is smooth compared to the shortest wavelength. This effective medium yields nearly the same deformation field as the initial detailed medium when subject to low-frequency wave propagation. The advantages of DWEU as a homogenization technique are two-fold: (i) the approach is conceptually simple and easy to program with basic linear algebra, and (ii) it yields explicit effective material profiles directly on a coarser grid. This is a big advantage as these effective, coarsened grids can be used in any numerical scheme to solve the wave equation more efficiently. Furthermore the method is not restricted to wave propagation but can be applied to any linear partial or ordinary differential equation.

The DWEU transforms the whole discrete wave equation into the spatial Fourier domain. Then an equation for the low-wavenumber component of the solution is derived, which is not yet a wave equation. For low temporal frequencies the high-wavenumber components are assumed to be nearly zero. We set them to zero and indeed retrieve an effective discrete wave equation for the low-wavenumber component. This effective discrete wave equation explicitly contains the effective material properties sampled on the coarsened grid. Interestingly, these material properties have non-local rheology, indicating that sub-wavelength structures and non-local rheology can have similar effects on the wave.

After a conceptual study in 1D (Fichtner & Hanasoge, 2017) and a study on the 2D Laplacian equation (Hansaoge, 2016) we present results for the elastic wave equation in 2D for both the SH- and P-SV cases.

Fichtner, A. and Hanasoge, S. M. (2017). Discrete wave equation upscaling, Geophys. J. Int, 209, 353–357.

Hanasoge, S. M. (2016). Spatio-spectral concentration of convolutions. J. Comp. Phys. 313, 674–686.