



Having the Right Reasons for Wrong Answers: An Inverse Solution for the 1-D Advection Diffusion Equation with Arbitrary Surface Boundary Conditions

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Two common refrains about using the one-dimensional advection diffusion equation to estimate fluid fluxes, thermal conductivity, or bed surface elevation from temperature time series in streambeds are that the solution assumes that 1) the surface boundary condition is a sine wave or nearly so, and 2) there is no gradient in mean temperature with depth. Concerns on these subjects are phrased in various ways, including non-stationarity in frequency, amplitude, or phase. Although the mathematical posing of the original solution to the problem might lead one to believe these constraints exist, the perception that they are a source of error is a fallacy.

Using a combination of laboratory experiments and numerical modeling, we validate a re-derived solution with an arbitrary surface boundary condition for temperature, demonstrating the frequency-independence of the inverse solution. This means that any single frequency can be used in the classical solutions for the thermal diffusivity and 1-D fluid flux in streambeds, even if the forcing has multiple frequencies. The practical consequence is that diurnal variations with asymmetric shapes, gradients in the mean temperature with depth, or 'non-stationary' amplitude and frequency (or phase) do not actually represent violations of assumptions, and they should not cause errors in estimates when using one of the suite of existing solution methods derived based on a single frequency. Misattribution of errors to these issues constrains progress on solving real sources of error. We apply the insights from the new solution to consider the utility of information at 'non-standard' frequencies and multiple frequencies to augment the information derived from time series of temperature.