LAGRANGE:
how to study the solar gravito-magnetism and internal dynamics by means of a Sagnac-like experiment at the scale of the Inner solar system

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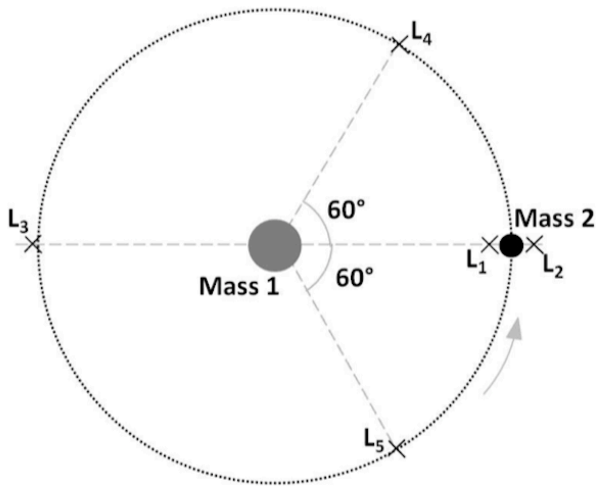
LAGRANGE: the Idea



We use spacecraft located at the Sun-Earth Lagrange points, as a physical reference frame and plan to perform time of flight measurements of electro-magnetic signals traveling on closed paths between the points. This allows to

- refine gravitational time delay knowledge in the field of the Sun and the Earth
- detect the gravitomagnetic frame dragging of the Sun
- check the possible existence of a galactic gravitomagnetic field and in particular for a dark matter component
- define a relativistic positioning system

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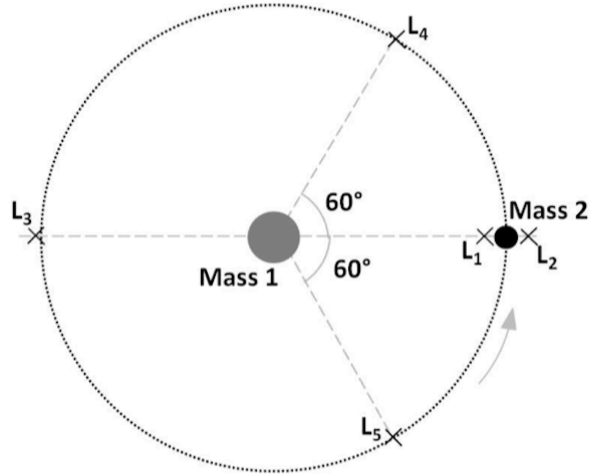
Gravitational time delay

A gravitational field produces a time delay on the propagation of electromagnetic waves. If the emitter position at is $y = -y_1$ and the receiver position is at $y = +y_2$, and b is the impact parameter the propagation time is

$$\Delta t_{prop} \simeq \overbrace{\frac{y_2 + y_1}{c}}^{\text{flat space}} + \underbrace{\frac{2GM}{c^3} \ln\left(\frac{4y_1y_2}{b^2}\right)}_{\text{mass correction}} \overbrace{\pm \frac{4GJ}{c^4 b}}^{\text{angular momentum correction}} + \underbrace{\frac{2GM}{c^3}\left(\frac{R}{b}\right)^2 J_2}_{\text{quadrupole correction}} + \dots,$$

Earth's quadrupole coefficient



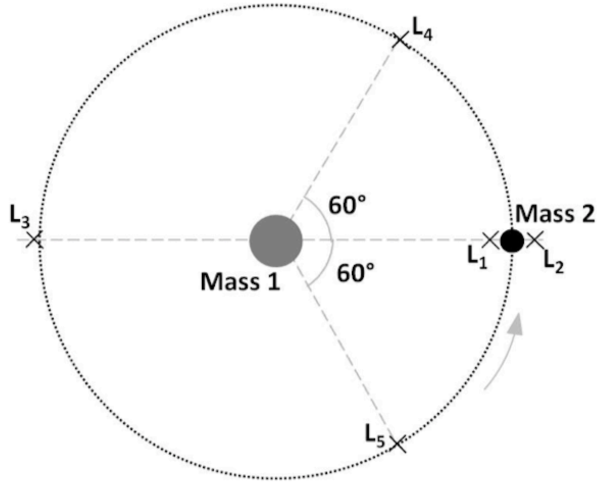
Propagation time measurements in the configuration L_1–Earth–L_2, allows (at least in principle) to obtain a measurement of the **Earth's quadrupole coefficient** in a way independent from the usual space geodesy techniques.
For instance, with an integration time of about 10^4 s it is possible to reach a precision in the measurement of the quadrupole coefficient of about $\delta J_2/J_2 \simeq 3 \times 10^{-8}$.

Frame Dragging

The **Lense-Thirring effect** or inertial frame dragging is the effect of the rotation of the source of the gravitational field; in particular, it is due to its angular momentum $\vec{J}$ and it is also called **gravitomagnetic effect.** The use of the Sun-Earth Lagrangian frame would allow a measurement of the solar Lense-Thirring effect exploiting a Sagnac-like approach based on the time of flight measurements.
The time difference between the right- and left-handed time of flight along the same elementary section of the path is

$$c\delta\tau = -4 \oint_{\text{path}} \frac{GJ}{c^3 r} d\phi$$

Frame Dragging



For instance, let us consider as closed path the triangular loop having L_4, L_2 and L_5 at the corners. The total expected time of flight asymmetry turns out to be

$$\delta\tau \simeq 4.30 \times 10^{-13} \text{ seconds}$$

which is well within the range of measurability, at least in terrestrial laboratory conditions. The challenge is to measure it in space.

Measurement of a possible galactic gravitomagnetic field

When addressing the effects of rotating massive bodies (Sun) on a local space-time geometry in our planetary system, it is rational to consider also possible analogous effects originating from larger structures dynamics, i.e. from our Galaxy or even more. The presence of a galactic gravitomagnetic field could be evidenced by the envisioned Lagrangian points configuration at the scale of an AU. The gravitomagnetic field might mimic the effects typically associated to the presence of **dark matter**: whence the interest in measuring a possible Lense Thirring effect.

Relativistic positioning system

An intrinsically **relativistic positioning system** is based on the local timing of at least four remote independent sources of electromagnetic pulses; emitters of regular pulses can be set in the Lagrangian points and will constitute the basis for a physical reference frame co-orbiting with the Earth

- see the talk *Fully relativistic positioning for the Galileo for Science (G4S) project* by David M. Lucchesi in session G1.3 - High-precision GNSS: methods, open problems and Geoscience applications.

Reference

A. Tartaglia, E. C. Lorenzini, D. Lucchesi, G. Pucacco,
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"How to use the Sun–Earth Lagrange points for fundamental physics and navigation,"
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