Ray tracing study of lower-band whistler mode emissions in outer radiation belts

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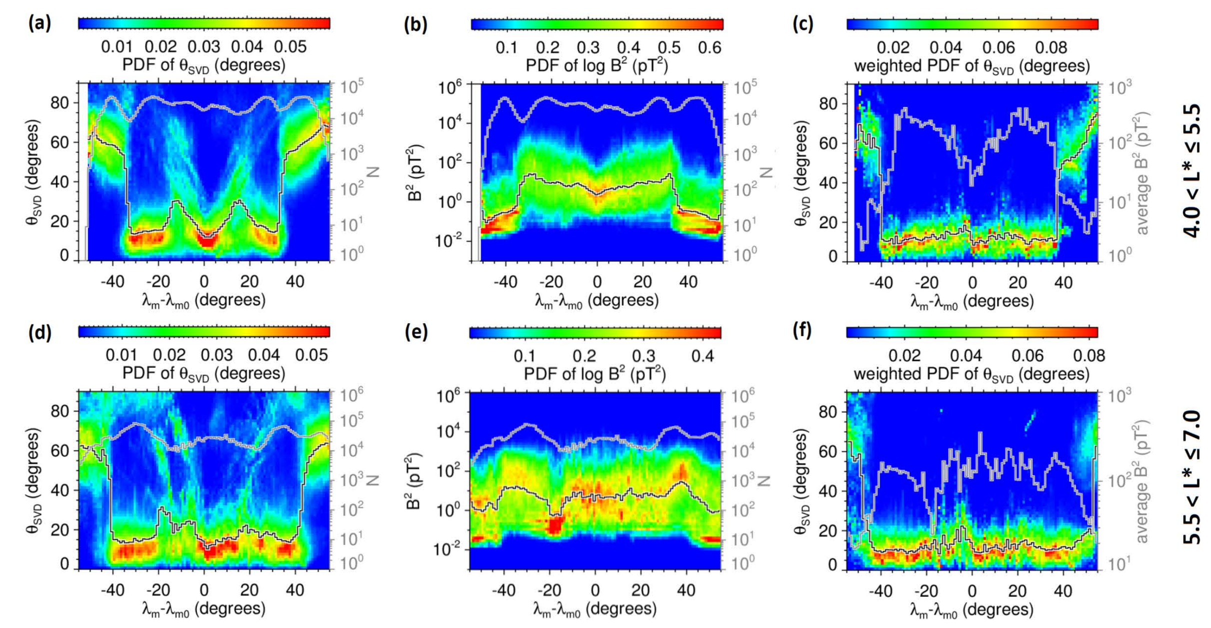
Introduction

- Statistical results of [1] based on **11 years of Cluster satellites measurements** show typical parameters of whistler-mode waves in range of equatorial frequencies from $0.1\omega_{ce}$ to $0.5\omega_{ce}$, for latitudes in the interval (-55°, 55°)
- The data were filtered so that they contain only waves with planarity greater than 0.5 and ellipticity greater than 0.2 (right-hand polarized). These values were computed by SVD methods from [2].
- We observe **increase of the wave magnetic energy B^2** in the lower interval L = (4.0, 5.5) and fluctuations of about one order of magnitude in the upper interval L = (5.5, 7.0)
- Wave normal angle θ_k is centered about 15°, with distinct increase of median values up to about 30° in range λ_m = (5°, 20°), more prominent in the lower L-interval
- By **weighting the θ_k distribution by B^2**, the increases of θ_k at lower latitudes almost disappear
- By the means of hot plasma ray tracing simulations , we aim to answer these questions:
  - a) Are the emissions **ducted or unducted?**
  - b) Can the dependence of θ_k on magnetic latitude be explained by **mixing of ducted and unducted waves?**
  - c) Can the dependence of **B^2 on latitude** be **explained by linearized hot plasma theory?** If not, to what extent?
  - d) What type of weighting is more natural: By the wave magnetic energy B^2 or by the wave energy density W?



a) *Probability density distribution function (PDF) of wave normal angle ϑ_k computed by the singular value decomposition (SVD) from 3B+2E orthogonal components of STAFF-SA data. Black solid line is the median value, grey solid line is the number of spectral matrices processed in each 1° bin of latitude. Latitudinal scale is centered around λ_{m0} which indicates the minimum of ambient magnetic field strength.*

b) *Wave magnetic energy B^2. Solid lines have the same meaning as described above.*

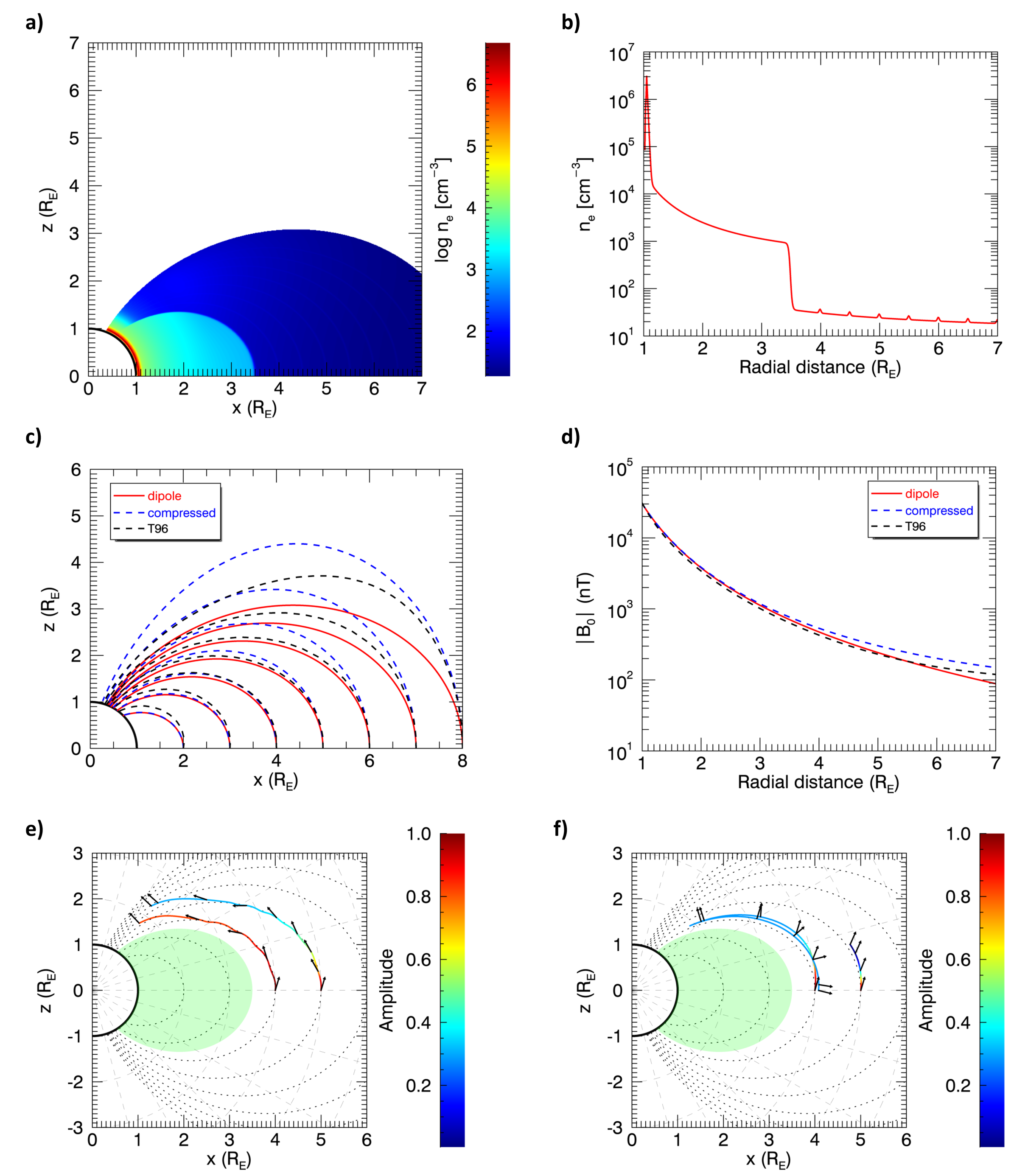
c) *Distribution of wave normal angles weighted by B^2. Solid grey line shows the average value of B^2.*

d-f) Same as a-c), but for waves in the upper interval L = (5.5,7.0).*

Methods, models and ray examples

- We use **3D ray tracing in a hot plasma** to simulate whistler mode waves originating at the equator propagating to the northern hemisphere. Since the propagation in longitudinal direction is not significant, we restrict our simulation to the meridional plane.
- Density model assumes an exponential decrease of equatorial density of electron-proton plasma with a latitudinal dependence based on semi-empirical model from [3]. Possible MLT dependence of density distribution is not included.
- Interaction with **plasmapause is not considered**, propagation stops at L <= 3.6
- Magnetic field is approximated by **a dipole and** then by a model of strongly **compressed magnetic field** from [4] to match some properties of better models like Tsyganenko96 + IGRF [5]
- **Initial θ_k are produced from Gaussian distribution** with a standard deviation of 12° and a mean value of 0° (taken from [1])
- To model hot plasma, we add **1% of 1 keV electrons** to a cold background of electrons and protons (1 eV)
- **Anisotropy**, defined as $a = \frac{T_{perp}}{T_{para}} - 1$, is taken to be constant everywhere with three different values: 0.0, 0.5 or 1.0
- **Ducting is modelled by Gaussian peaks in density** aligned to starting field lines, peak value 120 % of background density, σ = 100 km
- Wave energy density is calculated as $W = \frac{1}{16\pi}\left(B^* \cdot B + E^* \cdot \frac{\partial(\omega K)}{\partial\omega} \cdot E\right)$ where K is the dielectric tensor (hermitian)

- For each setting (magnetic field, hot/cold, anisotropy, ducting) we take 25 values of θ_k, 7 values of L (4.0 – 7.0, step 0.5) and 5 frequencies (ω/ω_{ce} from 0.1 to 0.5, step 0.1). Propagation is stopped at λ_{mag} = 55°, or at the equator after magnetospheric reflection (the case of unducted waves), or when amplitude is too small, $A^2 < 10^{-5}$
- During propagation we assume **conservation of Poynting flux** (before the inclusion of damping/growth)
- For ducted waves, increase of energy density due to **diverging or converging magnetic field** lines is taken into account
- For unducted rays, the increasing/decreasing distance of three rays (outermost minus innermost) with close starting points serves as the geometric factor; every unducted ray was therefore simulated thrice with difference in initial L-shell of 0.1



a) *Electron density distribution, limited to L = 8.*

b) *Equatorial profile of electron density with ducts.*

c) *Magnetic field lines of dipole, compressed dipole with B_{ext} = 0.002 B_{dip}, T96 for p_{dyn} = 6 nPa.*

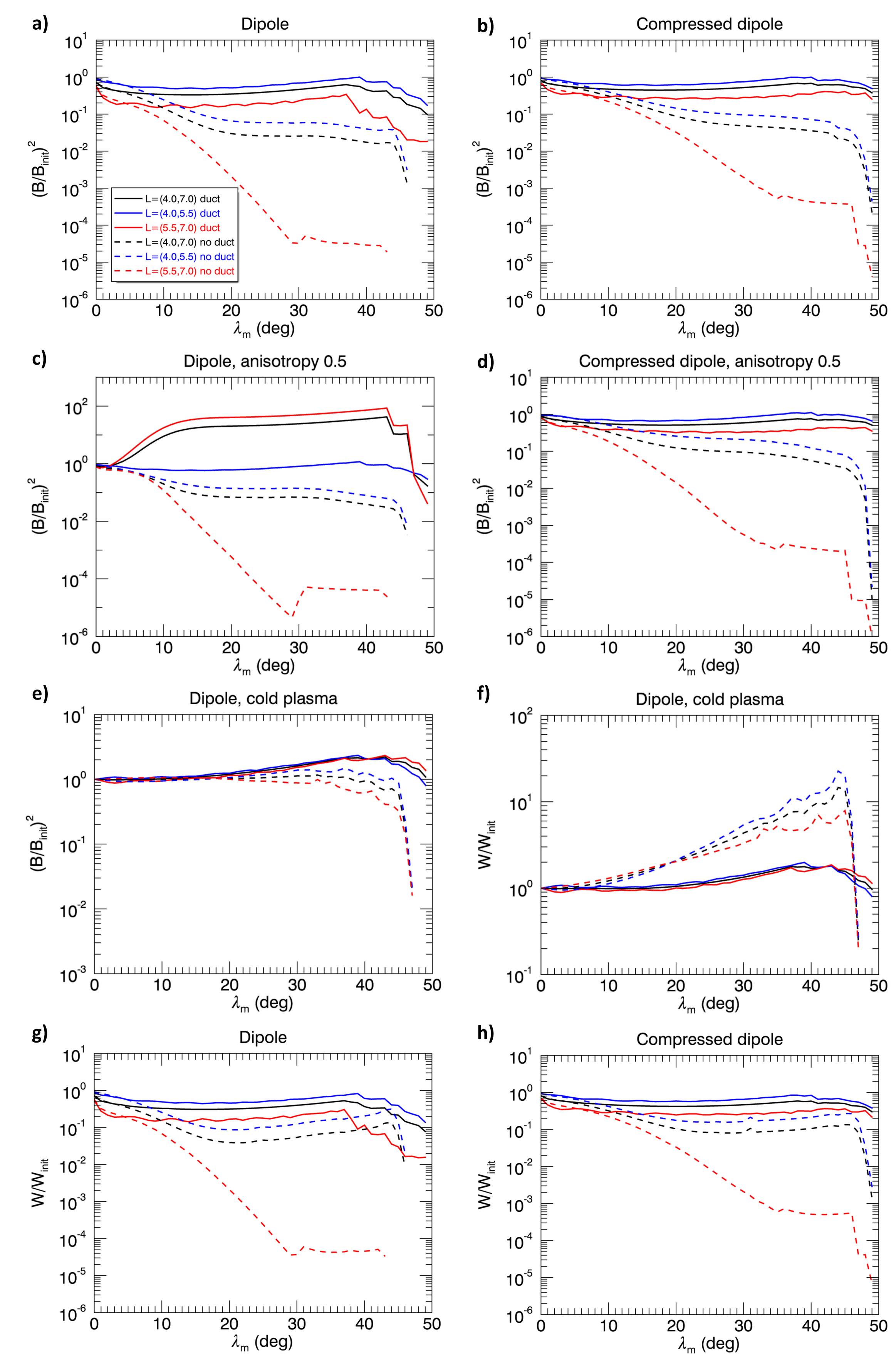
d) *Equatorial profile of magnetic field strength, same parameters as in c).*

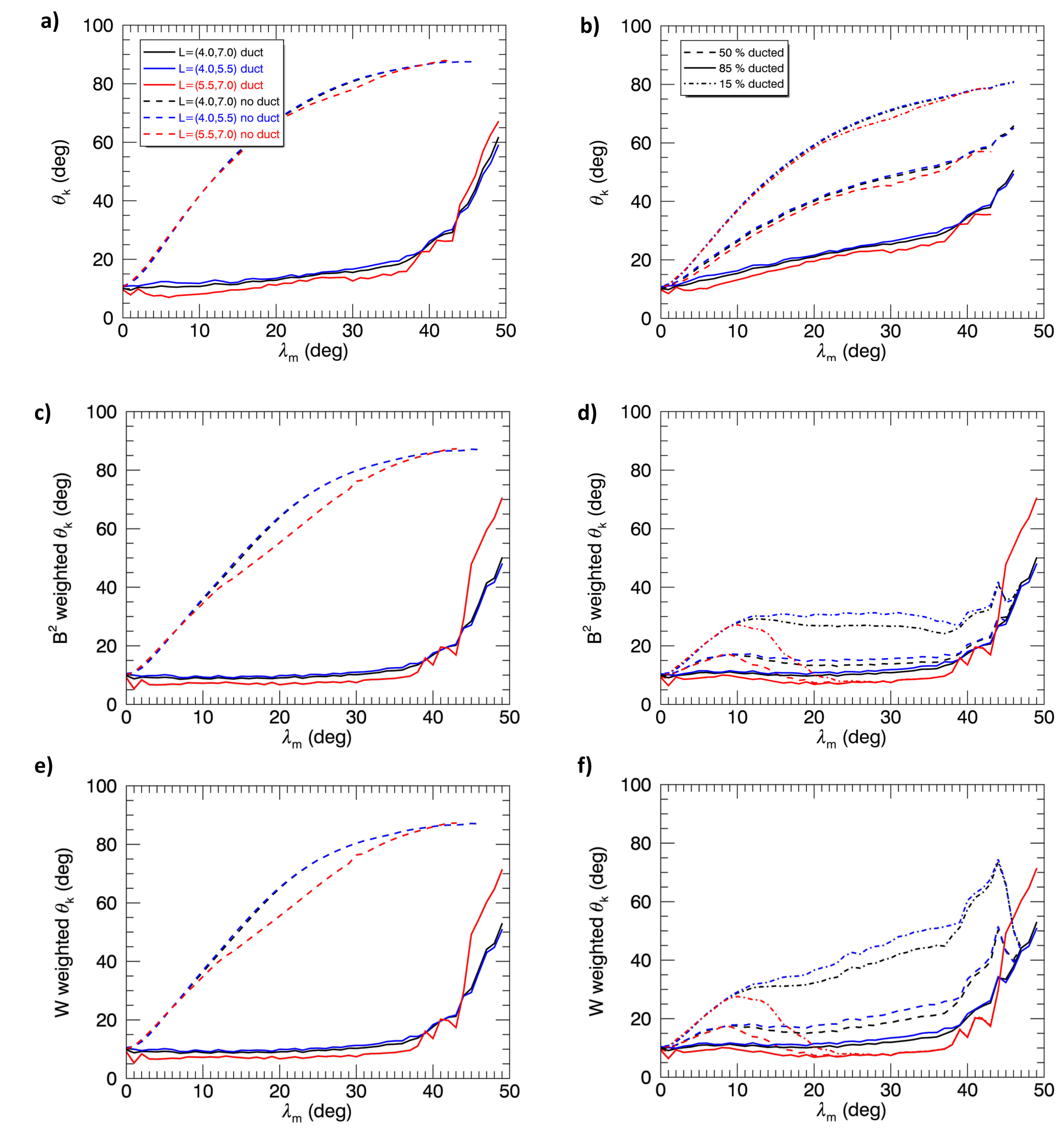
e) *Example of ducted rays with frequency f = 2 kHz, ϑ_{k0} = 16.6°, dipole model.*

f) *Example of unducted rays, same parameters as in e).*

Statistical results

- Ray paths continuing after **magnetospheric reflection** on ω_{lh} are **not included in the statistics** (discriminated by Poynting vector direction)
- Lower and upper L-intervals are treated both separately and together. Some fluctuations in separated data are due to waves traveling between the two intervals.





LEFT PANE: Solid lines show ducted propagation, dashed lines unducted. Red lines are higher L-shells, blue lines lower L-shells, black lines show results for all L-shells. Value at zero is obtained by averaging values at each computed trajectory point from interval λ_m = (0°, 1°), etc.

a-d) Magnetic wave energy B^2 normalized to initial value, a-b) without anisotropy, c-d) anisotropy 0.5.

e-f) B^2 and wave energy density W, cold plasma.

g-h) Wave energy density normalized to initial value.

RIGHT PANE: All dipole field. The left column shows wave normal angle for separated ducted and unducted waves, meaning of lines same as in the left panel (dashed unducted, solid ducted).

Right column shows mixing of ducted and unducted rays. Solid lines are 85 % ducted, 15 % unducted. Dashed lines are 50 % ducted, 50 % unducted. Dash-dotted lines are 15 % ducted, 85 % unducted.

a-b) ϑ_k taken as a simple average, i. e. assuming that each ray has the same weight.

c-d) ϑ_k weighted by the magnetic energy of rays.

e-f) ϑ_k weighted by the energy density. With the unducted portion increasing, substantial increase in ϑ_k is seen at higher latitudes, which is to be expected after inspecting figure g) in the left pane.

Conclusion

- Presented **increase in B^2** of ducted rays due to converging dipole field lines is **in qualitative agreement with the Cluster data**
- **Unducted rays** experience stronger damping than ducted rays and therefore **cannot answer for the increase in B^2**
- Assumption of heavily **compressed magnetosphere** (applicable to dayside only) naturally **leads to** even smaller growths or even **decreases in B^2**, but here sources in high latitude B_{min} pockets can be considered
- Considerable increase in wave energy density is observed in unducted waves due to decreasing group velocity. Weighting of θ_k by W leads to increase of the angles at higher latitudes, which is not observed in the data.
- When we assume that a significant portion of waves coming from source is unducted, B^2 weighting of θ_k shows a bump at lower λ_m. This is not seen in the experimental data, which means that the **waves are mostly ducted** or that there is a mechanism that increases wave energy of quasiparallel waves.
- Uniform distribution of frequencies in not supported by the measured data and might lead to distortion of hot plasma effects, especially with anisotropy
- Temperature anisotropy affects the wave energy greatly even for reasonably low values. However, the simplistic model of constant anisotropy predicts the real situation very poorly. A model based on particle simulation and/or statistical data, e. g. [6], is needed to obtain plausible results.
- In summary, linearized hot plasma theory seems not to be completely sufficient to explain our measurements, as explained by [7], but some partial understanding might be achieved with better models

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