Processing strategies for comparisons of absolute meters

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Abstract

Starting in 1981, absolute gravimeters (AG) have been compared on a regular basis at international level. As a consequence of the Mutual Recognition Arrangement of the International Committee for Weights and Measures (CIPM MRA), the international comparisons of AGs are today split into key comparison (KC) and pilot study (PS), considering different subsets of meters and different contributions to the comparison reference values. Furthermore, the mean reference level was estimated independently for each epoch at the same station without assessing the plausibility of changes. Also the adjustment procedure changed over time, starting from different strategies for the homogenization of the instrument heights and reaching up to uncertainty estimates, weighting schemes and consideration of systematic errors. With the establishment of a new absolute gravity reference system, the international comparisons will gain importance as a backbone of its realization. We present a reprocessing of the recent comparisons, considering different processing approaches and showing differences between KC vs. KC+PS solutions, equal vs. weighted constraints, least squares vs. L1 norm solutions and to raise the question of how to obtain reasonable uncertainty estimates directly from the adjustment.

Introduction

A **combined** (observation and constraint equations) least squares adjustment has to be performed to determine:
- comparison reference gravity values (CRVs) at the stations
- biases of absolute gravimeters (AGs)

Inputs the *g*-values transferred to the reference comparison height and their associated uncertainties (u). Every measurement made by the gravimeter *"i"* (with a bias δ_i) at the station *"j"* during the comparison may be described by the observation equation

$g_{ij} = g_j + \delta_i + \varepsilon_{ij}$ $\Rightarrow$ design matrix **A** and observation vector ***l***

with respective weights w_{ij} ($w_{ij} = u_o^2/u_{ij}^2$ where u_o is the unit weight) $\Rightarrow$ standardly a diagonal (no correlations between all the measurements are taken into account) weighting matrix **P**

As the set of observation equations has no unique solution, a **constraint** which can be interpreted as **definition of the CRVs** is required:
- **non-weighted constraint:** $\Sigma\, \delta_i = 0$ applied in ECAG2011, ICAG2013
- **weighted constraint:** $\Sigma\, w_{\delta i}\, \delta_i = 0$ applied in ICAG2009, ECAG2015

$\Rightarrow$ matrix of constraint **B** that should represent the accuracy of gravimeters

This constraint defines the mean absolute level of the comparison!

Therefore the definition of weights for biases ($w_{\delta i}$) is a very important step and have to be derived based on correct uncertainty estimates of AGs. On the other hand **P** plays only a role of **relative weighting between AGs** and influences the determination of relative ties between stations but not directly the absolute reference level of the comparison.

CRVs and biases (vector ***x***) are obtained by solving the normal equations:

$$\begin{pmatrix} A^T P A & B \\ B^T & 0 \end{pmatrix} \begin{pmatrix} x \\ k \end{pmatrix} + \begin{pmatrix} A^T P\, l \\ 0 \end{pmatrix} = 0$$

References

Jiang et.al. (2011): The 8th International Comparison of Absolute Gravimeters 2009: the first Key Comparison (CCM.G-K1) in the field of absolute gravimetry, Metrologia, 49(6) 666-684

Francis et.al. (2013): The European Comparison of Absolute Gravimeters 2011 (ECAG-2011) in Walferdange, Luxembourg: results and recommendations, Metrologia, 50(3) 257-268

Francis et.al. (2014): Final report of the CCM.G-K2 Key Comparison

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Motivation and the re-processing

Demonstration and discussion how particular steps of data processing influence the results of comparison. The demonstration has been provided based on re-processing of 4 comparisons carried out in 2009, 2011, 2013 and 2015 (see references).

Following solutions are presented which are based on all observations, but differ in the way, how two groups of AG contribute to the reference level:

KC_n – only NMI/DI gravimeters have been **used in the non-weighted constraint**. Other gravimeters contribute only with gravity differences.

KC_w – as above, but the NMI/DI gravimeters are used in the **weighted constraint**, where weights are related to declared uncertainties of gravimeters

KC_wh – as above, but **uncertainties of gravimeters are harmonized**: Gravimeters declaring an uncertainty better than 2.4 µGal were changed to this value.

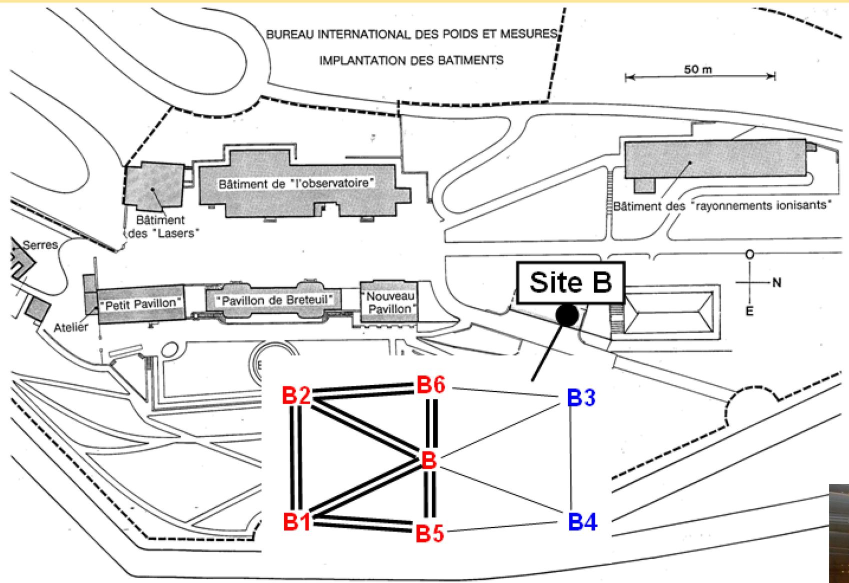
KC_who – as above, with **outlier detection** based on the consistency check (discussed below).

ALL_who – **all gravimeters** taken into account within the **weighted constraint** with harmonized uncertainties and outliers detection.

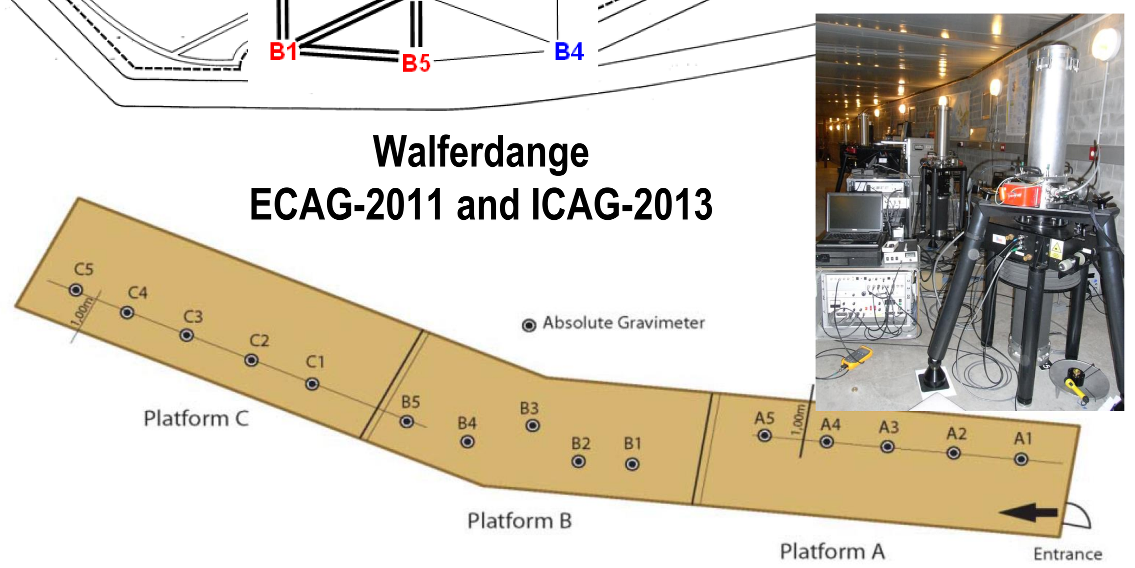
ALL_who_L1 – a **constraint that minimizes the L1 norm** of biases instead of imposing zero mean of biases.

ALL_wh – as **ALL_who** but without outlier detection.

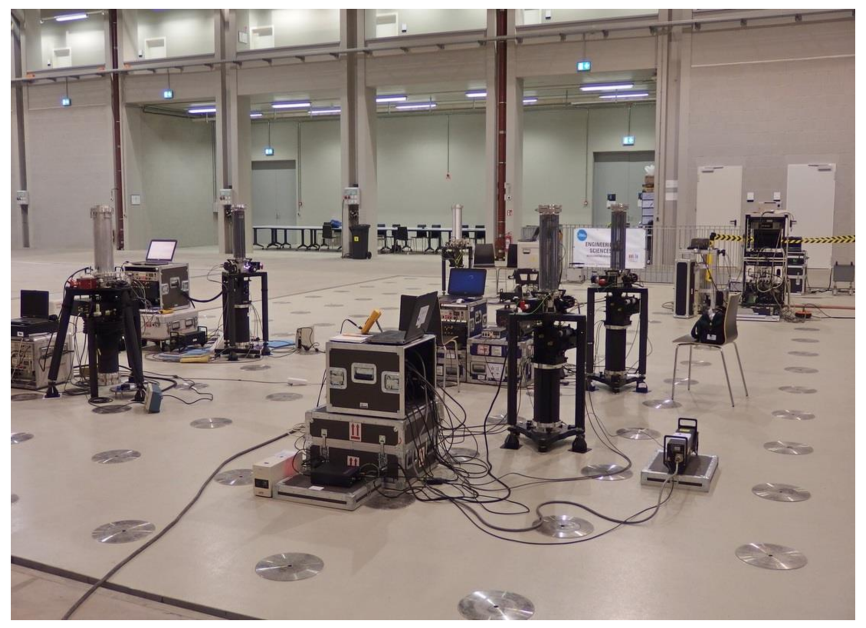
ALL_w – as **ALL_wh** but without harmonization of uncertainties.



Sévres ICAG-2009



Walferdange ECAG-2011 and ICAG-2013



Belval ECAG-2015

Results of the reprocessing

Comparison & Site	ICAG-2009 Sévres	ECAG-2011 Walferdange	ICAG-2013 Walferdange	ECAG-2015 Belval
# Station	5	15	15	9
# AGs KC / ALL	11/21	6/22	10/25	4/17
# FG5s KC / ALL	7/14	4/18	7/19	3/15
Step by step change of **the mean CRVs in µGal** (maximum differences in brackets)				
KC_n	Initial solution			
KC_w	**-1.02** (-1.03)	-0.50 (-0.50)	-0.60 (-0.60)	**-3.48** (-3.48)
KC_wh	-0.01 (0.13)	+0.04 (0.31)	-0.01 (-0.17)	+0.02 (0.36)
KC_who	-0.04 (0.57)	0.00 (0.00) No outlier	-0.14 (0.85)	-0.07 (0.68)
ALL_who	+0.72 (0.72)	-0.10 (-0.48)	+0.57 (0.68)	**+0.99** (1.33)
ALL_who_L1	-0.10	+0.20	-0.73	+0.88

All solutions are compared with respect to the **contribution** of the individual AGs **to the reference value**, i.e. to the condition equation.

The **non-weighted constraint** does not account for the declared uncertainties and **should not be used**. The reference level can be significantly biased in case of unequal uncertainties of the AGs, in particular when only a minor number of AGs are included. This can be seen in ECAG-2015, where the CRV is distorted by one of the 4 NMI/DI AGs having a significantly higher uncertainty that is not reflected within the non-weighted constraint.

At a first glance, the **harmonization** of uncertainties (maximum change of 1.8 µGal to 2.4 µGal) has only minor impact, since the largest change in the CRV is below 0.1 µGal. Nevertheless, in case of several AGs with low declared uncertainties, **measurements** can be **excluded**, based on the following **consistency check**

$$E_n = \frac{(g_{ij} - g_j)}{\sqrt{u^2(g_{ij}) + \sigma^2(g_j)}}$$

representing the ratio between the difference of measured and estimated reference gravity values (residuals) and the uncertainty of this difference, where the following contributions are included:

$u(g_{ij})$ .. uncertainty of the g-values at the comparison height,

$\sigma(g_j)$ .. standard deviation of the CRV at the station *j* obtained from the LSQ,

An absolute value of E_n **larger than 2** indicates that both gravity values are **incompatible at 95%** confidence level as their difference cannot be covered by their uncertainties.

Similarly, the **outlier detection** causes only small changes in CRVs. However, often measurements **from non-NMI/DI** AGs are excluded, which do not contribute to the definition of CRVs for KC solutions. In case of equal treatment of all gravimeters (ALL solutions), the situation is different, even though it is better determined due to larger number of AGs.

Difference in the mean CRVs (ALL_who w.r.t ALL_wh) in µGal			
ICAG-2009	**ECAG-2011**	**ICAG-2013**	**ECAG-2015**
+0.88	+0.35	+0.00	+0.34

Splitting the **group of gravimeters** to NMI/DI and non NMI/DIs influenced the comparison reference values at the level of 1 µGal. Further, the following number of measurements are inconsistent at a 95% confidence interval.

Number of outliers for ALL_w solution NMI/DI --- non-NMI/DI			
ICAG-2009	ECAG-2011	ICAG-2013	ECAG-2015
1 --- 4	0 --- 7	2 --- 4	1 --- 4

Finally, we tested a solution with **constraining L1 norm** of biases instead of zero mean of biases. We are looking for such a δ_c for which:

$$\sum_{i=1}^{n} |\delta_i + \delta_c| = \min \quad \text{or} \quad \sum_{i=1}^{n} |w_i\,(\delta_i + \delta_c)| = \min$$

In case of the L1 norm with non-weighted constraint, there was a **problem to detect an unique solution** for the parameters, since changes of the L1 norm were only minimal for a range of bias shifts of several microgals. Considering a weighted L1 norm approach is at least worth to deal with.

Correlations between gravimeters

Two types of correlations between AGs should be taken into account:
1) correlations between measurements of a **particular gravimeter**,
2) correlations between measurements of the **same type of gravimeters**.

As it is problematic to determine the second, the first type can be empirically obtained from the **repeatability** σ (random errors *only*) and the **uncertainty** u (including random *and* systematic errors) of an AG. Typically, the FG5(X) gravimeters have an uncertainty ≈ 2.4 µGal, while the repeatability, computed e.g. from the dispersion of measurements of a particular FG5 reaches ≈ 1.2 µGal. That means a **strong correlation of** r **=** $(2.4^2 - 1.2^2) / 2.4^2 =$ **0.75** between measurements of a particular AG, that can be easily reflected in the weighting matrix **P**. CRVs and biases are practically independent on the choice of *r*. However, the **error estimates are significantly different**:

$$r_{i,k} = \frac{s_{i,k}^2}{s_i s_k} = \frac{(u^2 - \sigma^2)}{u^2}$$

ECAG-2015, a posteriori σ for biases				
AGs	KC_who		ALL_who	
	r=0.00	r=0.75	r=0.00	r=0.75
FG5X-221	0.6	1.4	0.6	1.6
FG5-215	0.5	1.5	0.5	1.6
IMGC-02	2.6	6.1	2.6	5.9
FG5X-216	0.5	1.3	0.6	1.6
FG5X-102	0.8	2.0	0.6	1.6
FG5-202	0.8	2.0	0.6	1.6
FG5-218	0.8	2.0	0.6	1.6
FG5X-220	0.7	2.0	0.6	1.6
FG5X-229	0.8	2.0	0.7	1.6
FG5-230	0.8	2.0	0.8	1.7
FG5-233	0.8	2.1	0.7	1.7
FG5-234	0.8	2.0	0.7	1.6
FG5-238	1.9	4.5	1.7	4.2
FG5X-247	1.3	2.6	1.2	2.4
FG5-301	0.7	2.0	0.6	1.6
FG5X-302	0.6	2.0	0.5	1.6
A10-020	1.7	4.5	1.6	4.1

ICAG-2013, a posteriori σ for biases				
AGs	KC_who		ALL_who	
	r=0.00	r=0.75	r=0.00	r=0.75
A10-006	4.1	12.5	4.0	12.4
A10-020	2.1	6.3	2.1	6.1
CAG-01	2.5	6.3	2.5	6.3
FG5-102	1.3	3.1	1.3	3.0
FG5-202	1.1	3.0	1.1	2.8
FG5-206	1.1	3.0	1.1	2.8
FG5-213	1.1	2.8	1.1	2.9
FG5-215	1.0	2.6	1.0	2.8
FG5-218	1.1	3.0	1.0	2.8
FG5-223	1.2	3.0	1.1	2.8
FG5-228	1.1	3.0	1.0	2.8
FG5-231	0.9	2.3	1.0	2.8
FG5-233	1.1	3.0	1.0	2.8
FG5-234	1.1	3.0	1.0	2.8
FG5-242	1.8	3.3	1.9	3.4
FG5-301	1.1	3.0	1.0	2.8
FG5X-104	0.9	2.2	1.0	2.8
FG5X-209	0.9	2.3	1.0	2.8
FG5X-216	1.1	3.0	1.0	2.8
FG5X-220	1.1	3.0	1.0	2.8
FG5X-221	1.0	2.6	1.1	2.8
FG5X-302	1.2	3.1	1.1	2.8
IMGC02	2.0	6.1	2.0	6.1
NIM-3A	2.0	5.7	2.0	5.7
T-2	2.4	6.1	2.4	6.0

Error estimates from the adjustment of the comparison with r = 0 cannot be seriously used, neither for outlier detection nor for estimation of a posteriori uncertainties. This should be solved by including correlations within the weighting matrix.

Conclusions

For the evaluation of comparison of AGs it is recommended to
- use a weighted constraint to fix the comparison reference values, realistic uncertainty estimates assumed
- harmonize the uncertainties of the AG resp. introduce realistic estimates
- include the correlation for a particular AG in order to obtain realistic error estimate after adjustment

Significant changes of the CRV may be results of the selection of a subset of AGs (KC), while the detection and removal of outliers had only minor impact in the CRV for the analyzed solutions.