



A structure-preserving split finite element discretization of the rotating shallow water equations in split Hamiltonian form

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We introduce locally conservative, structure-preserving split finite element (FE) discretizations of a y -independent (slice) model of the covariant rotating shallow water equations. Studying this y -independent model case provides insight towards developing schemes for the full 2D rotating shallow equations and more sophisticated models. These split schemes are derived using the split Hamiltonian FE method, which is an extension of split FE framework of [1]. The splitting introduces two chains of compatible FE spaces such that the differential operators hold strongly. This leads to a separation of the set of equations into topological prognostic and metric-dependent closure equations. Consequently, the structure preservation is related to the topological equations, namely the conservation of the Hamiltonian (i.e. energy) follows from the antisymmetry of the Poisson bracket and the conservation of mass, potential vorticity and potential enstrophy from the bracket's Casimirs. These structure-preserving properties are not affected by the metric closure equations that, in turn, carry metric information and are therefore responsible for the schemes' accuracy, stability, convergence and discrete dispersion properties. We verify the clear separation of structure preservation and numerical properties analytically and show numerical examples of geophysical relevance for a low order single mesh implementation.

References

- [1] Bauer, W. and Behrens, J. [2018], A structure-preserving split finite element discretization of the split wave equations. *Applied Mathematics and Computation*, **325**, 375–400.