

Global random walk solutions for flow in porous media

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The elliptic equation governing the pressure head for stationary flows in porous media has the structure of a diffusion equation and can therefore be solved by particle methods. The straightforward approach would be as follows: express the Stratonovich diffusion operator in the original equation as sum between the Fokker-Planck operator and a drift term consisting of the row derivative of the coefficient tensor, generate trajectories of the Itô equation associated to the non-stationary Fokker-Planck equation by tracking computational particles in continuous space, and finally estimate the pressure head by the density of the system of particles in the limiting stationary state. The alternative approach proposed here starts with the staggered finite difference scheme of the non-stationary equation in Stratonovich form, from which one derives biased random walk rules governing the movement on a regular lattice of the system of particles used to obtain a numerical approximation of the solution. The procedure is implemented as a Global Random Walk (GRW) algorithm which moves simultaneously all the particles from a lattice sites to neighboring sites, according to the local jump probability. Consequently, the algorithm is orders of magnitude faster than classical sequential particle tracking approaches. Moreover, the GRW procedure can be applied to arbitrarily large numbers of particles, which ensures highly accurate numerical solutions. Since the number of computational particles is conserved, the algorithm is stable and well suited for simulating flows in heterogeneous porous media. The GRW approach is validated in the one-dimensional case by comparisons with analytical solutions of the flow equation with rapidly oscillating coefficients. The proposed numerical approach can be further developed into an integrated GRW solution for flow and solute transport which avoids interpolation errors, inherent when flow solutions obtained by other numerical methods are used in GRW transport simulations. The drawback of the GRW flow solver is the transitory time needed to reach the steady state, which renders it about ten times slower than a finite element solver. However, since the time needed to import velocity fields provided by external solvers into the GRW lattice has been found to be two orders of magnitude larger than the time taken by the GRW transport simulation itself, one expects that the integrated approach could lead to important saving of computing time.