



Sequential Monte Carlo for Stochastic Advection by Lie Transport (SALT): A case study for the damped and forced incompressible 2D stochastic Euler equation

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We present an effective high dimensional data assimilation methodology for the damped and forced incompressible 2D Euler fluid flows driven by stochastic advection by Lie transport (SALT) type noise. SALT type stochastic partial differential equations (SPDE) for fluids were introduced by [Holm, Proc Roy Soc, 2015]. According to [Holm, Proc Roy Soc, 2015] and [Cotter et al., 2017], the principles of transformation theory and multi-time homogenisation imply a physically meaningful, data-driven approach for decomposing the fluid transport velocity into its drift and stochastic parts, for a certain class of fluid flows. This results in a stochastic parameterisation for the fluid transport and gives us SPDE models for the fluid flows which preserve circulation.

A numerical methodology for implementing this velocity decomposition was developed in [Cotter et al., 2018] and applied to the 2D Euler equations, on a simply connected domain with no-penetration type bc, in consideration

$$d\omega + \left(\mathbf{u}dt + \sum_i \xi_i \circ dW_t^i \right) \cdot \nabla \omega = f(\omega).$$

Successful uncertainty quantification results were obtained thus forming the ground work for this data assimilation study. Our SPDE model is prescribed on coarse resolutions, whereas the “original” deterministic partial differential equations are prescribed on fine resolutions. We consider two stochastic filtering data assimilation problems. In the first problem the observations come from a single realisation of the SPDE. In the second problem the observations correspond to the fine scale PDE system.