



A simple stream function-vorticity formulation of mixture mass flows

Puskar R. Pokhrel (1,2), Khim B. Khattri (1), and Shiva P. Pudasaini (3)

(1) School of Science, Department of Natural Sciences, Kathmandu University, Kavre, Nepal (prpokharel@student.ku.edu.np), (2) Department of Mathematics, RR Campus, Tribhuvan University, Kathmandu, Nepal, (3) Institute of Geosciences and Meteorology, Geophysics Section, University of Bonn, Germany

Geophysical mass flows such as landslides and debris flows contain the mixture of soil and rock particles with a significant to dominant quantity of viscous fluid. Employing the general two-phase mass flow model (Pudasaini, 2012), recently Pokhrel et al. (2018) developed a generalized quasi-two-phase bulk mixture model which includes effective mixture viscosity that evolves mechanically as a coupled function of several physical and mechanical parameters and dynamical variables. The later model has further been formulated in a new stream function-vorticity frame (Pudasaini and Pokhrel, 2019). The original system of partial differential equations in velocity and pressure has now been converted in to the stream function-vorticity form as a close system of equations that is free of pressure term. Thus, the original three equations have been replaced by a set of just two partial differential equations. So, the new systems is advantageous. Importantly, a novel pressure Poisson equation in terms of stream function, vorticity and the rate-dependent mixture viscosity is derived. The two equations are coupled through the Poisson equation. This, then, provides the full system. However, the pressure is decoupled and can be computed separately with the knowledge of stream function and vorticity. This is favorable over the original system. We further reduced the new system and obtain exact expression for stream function. The most important aspect is the construction of a new pressure Poisson equation for shear mixture flow that includes yield strength of the mixture. The importance of the pressure Poisson equation induced by the flow field intensity, the yield strength, and free surface geometry are discussed. The flow depth and the non-linear diffusion of the free surface play crucial role in characterizing the pressure Poisson equation. Mixture pressures are derived analytically for thin and thick flows. Similarly, pressure dominated flow; thick, low yield strength flow; and thin, high yield strength flows are analyzed. Moreover, based on the developed pressure Poisson model, several exact analytical solutions are constructed for the pressure and flow depth distributions for incipient flow, shearing flow, propagating bore front and mass deposition. Presented novel models and analytical results are in line with observed phenomena and highlight the application potential of the new model in relatively easily describing the mixture flow dynamics as compared to the original complex models.

References:

- Pokhrel, P. R., Khattri, K. B., Tuladhar, B. M., and Pudasaini, S. P. (2018): A generalized quasi two-phase bulk mixture model for mass flow. *Int. J. of Non-Linear Mech.*, 99, 229-239.
- Pudasaini, S. P., Pokhrel, P. R. (2019): A stream function-vorticity formulation of a general mixture mass flow model. (In preparation).
- Pudasaini, S. P. (2012): A general two-phase debris flow model. *J. Geophys. Res.* 117, F03010, 10.1029/2011JF002186. doi: 10.1029/2011JF002186.