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On a relativistically invariant wave-current formulation

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A peculiar feature of the wave-current problem is the inconsistency in the dispersive properties between Lagrangian and Eulerian descriptions. In the former, frequency-wavenumber dispersion is single-valued and isotropic. Whereas, it is multivalued and anisotropic in the latter (Peregrine, 1976). This appears to be a paradox and wave models and measurements tend to disregard the additional modes. In fact, in other fields involving dispersive waves in moving media they are known solutions called “velocity-induced-modes” (v.i.m.) (Censor, 1980). Within the water wave context, they may occur in the gravity and gravity-capillary regimes and may introduce new nonlinear interactions (Kouskoulas and Toledo, 2017) which can effect Bragg scattering and radar oceanography.

In the present work, the apparent paradox of v.i.m. for water waves on current is resolved using a relativistic approach. The mathematical tools have been discussed within the context of electrodynamics (McCall and Censor, 2007), plasma waves (Censor, 1980), and aeroacoustics (Gregory et al., 2015). The approach employs Lorentz transformations to rewrite the nonlinear boundary value problem for surface gravity-waves on uniform current in flat (Minkowski, 1908) spacetime. Potential flow is assumed and a new linear solution and spacetime dispersion relation is derived.

The generality of the spacetime dispersion relation supersedes the commonly used Galilean formulation which is only locally invariant. Accordingly, well known gravity wave dispersion relations for Eulerian and Lagrangian observers are recovered through application of the appropriate limits. Thus, the spacetime dispersion relation provides a physically consistent conceptual and mathematical bridge between Eulerian and Lagrangian descriptions. The dispersion relation for various current magnitudes is shown in Euclidean space and Minkowski (flat) spacetime. In Minkowski spacetime, with increasing current, the “growth” of the v.i.m. from zero is seen.

In addition, the relativistic approach permits for correct calculation of group velocities for all four wavenumber modes, which cannot be achieved with the non-relativistic approach. Moreover, in spacetime geometry for general mechanics, c (conventionally called the speed of light) plays the role of a limiting velocity (Minkowski, 1908). The limit c→1 is used to show dispersion near singularities (i.e. critical layers). The relativistic approach can be extended to investigate nonlinear interactions between waves with a wide disparity scales, for which existing approaches would introduce considerable inaccuracies.