



## **Variational Inequality Free Surface Motion of a Glacier Surface with Mass Balance**

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Simulating glacier surface evolution based on surface velocity and surface mass balance estimates is a common task in glaciology. Like any gravitationally driven flow that is not constrained at the upper surface, glaciers and ice sheets feature a free surface which becomes a free boundary problem within simulations. A kinematic boundary condition is often successfully utilized to represent the surface. However the naturally occurring constraint that the surface elevation ( $S$ ) can not fall below the bed elevation ( $B$ ), ( $S - B \geq 0$ ) in combination with a non zero mass balance complicates matter substantially.

We present a numerical model to simulate the free surface evolution of glaciers that directly incorporates this natural constraint. It is based on the finite element software package FEniCS solving the Stokes equations for ice flow and a transport equation for the free surface evolution. The evolution of the free surface is treated as a variational inequality, constrained by the bedrock underlying the glacier or the topography of the surrounding ground. To solve this problem, the 'constrained' non-linear problem solving capabilities of PETSc's SNES interface are used. We partition the computational domain into regions where ice exists and its ice-free surroundings. For ice regions we solve the constrained kinematic boundary condition including mass balance and for the surroundings we only solve the constrained mass balance term. This approach is mass conserving as the constraint is considered in the solving process and thus does not require any ad-hoc post-processing steps to enforce mass conservation. The model will become freely available in the framework of the `debadvect` model (<https://github.com/awirbel/debadvect>), also providing additional functions for the generation of updated 3D domain models.