



## **Ionosphere electron density modeling using the regularized constraint optimization approach**

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The Earth's ionosphere is classified into discrete layers, namely D, E, F1, F2 and the plasmasphere based on the altitude. The spatial part of the electron density within the ionosphere is a three dimensional parameter varying with latitude, longitude and altitude. Accordingly, the electron density can be modeled as the sum of the electron densities of the individual layers. In this paper, we present details of the global electron density modeling problem using two-dimensional uniform B-Splines. The Chapman profile function is chosen to define the electron density along the altitude. The key parameters, i.e. the peak electron density, the peak height and the scale height for all the ionosphere layers, are modeled by series expansions in terms of B-spline basis functions. To be more specific, we use polynomial B-splines for latitude and trigonometric B-splines for longitude modeling. This way, the electron density modeling is set up as a parameter estimation problem.

In this paper we describe the procedure for a simultaneous estimation of B-spline series coefficients related to the complete set or – as more realistic – to a subset of ionospheric key parameters. The main focus of this paper is two-fold. First we analyze the estimated B-spline series coefficients for the selected key parameters with regard to the imposition of absolute value bounds in the form of inequality constraints. The convex optimization approach is used for the estimation of the unknown B-spline series coefficients. A set of true B-spline series coefficients defined on a  $5^\circ \times 5^\circ$  global grid is given as reference for verification and comparison purposes.

Secondly, we describe the use of two different regularization techniques corresponding to the minimization of the L1 and the L2 norm objective cost function. Therefore, this paper provides a discussion of the impact of inequality constraints and the choice of regularization techniques on the modeling accuracy. We analyze the different correlations between the estimated B-spline coefficients among the selected set of key parameters. Therefore a judicious choice of the key parameters set allows for the analysis of specific layers of the ionosphere. We describe the necessary and sufficient conditions for the existence of a solution and also show how these two aspects affect the overall feasibility and stability of the optimization solution to the problem. We describe the importance of the convexity check of the optimization problem and evaluate the Karush-Kuhn-Tucker (KKT) conditions alongside the optimized solution. We use the IRI model and electron density measurements from COSMIC and CHAMP as input data. We also analyze the statistics of the estimated electron density key parameters and the residual sum of squares of the optimization cost function.