



Discrete structure conservation for reversible and irreversible motions in geophysical fluid dynamics

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For many applications, it is desirable numerical models of geophysical fluid dynamics are structure-preserving: they possess discrete processes and properties that closely mimic those of the continuous equations. For example, considering large-scale atmospheric dynamics these might be balanced states and adjustment processes, wave motions (Rossby, Kelvin, and Inertia-Gravity) and conservation properties (total mass, energy, etc.). Underlying these properties for reversible (entropy-conserving) dynamics at the continuous level is the Hamiltonian formulation, which writes the equations of motion in terms of a Hamiltonian function and a Poisson bracket. A general framework for structure-preserving discretizations of reversible dynamics then consists of discretizing the Hamiltonian formulation using a mimetic spatial discretization and an energy-conserving Poisson integrator to produce a quasi-Hamiltonian discrete model. This is sufficient to obtain most of the desirable properties, and we will show that careful selection of the spatial and temporal discretization gives rest. Specifically, we have chosen to use the newly developed mimetic Galerkin difference element (MGD, a type of compatible Galerkin method) coupled with a second-order, implicit energy-conserving Poisson integrator. The MGD element avoids spectral gaps and other dispersive anomalies found with finite elements methods. Using these choices, concrete examples will be shown of the application of the general framework for two commonly used sets of equations in geophysical fluid dynamics: the thermal shallow water equations and the fully compressible Euler equations. Results from commonly used test cases such as planar versions of the DCMIP test suite will be shown. In both cases, for the first time, models with fully discrete conservation of total mass, buoyancy or entropy and energy for arbitrary equations of state are obtained. If time permits, there will also be a short discussion of ongoing working on the extension of the general framework to incorporate irreversible (entropy generating) processes, including subgrid turbulence parameterizations.