Model selection and generation of bias in small samples of the earthquake frequency-magnitude distribution

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The Gutenberg-Richter (GR) b-value controls the relative weighting of small and large earthquakes in a population and hence is an important parameter for the assessment of earthquake hazard. For tectonic seismicity, it is often close to unity, but some studies have shown the b-value to be elevated (>1) in both volcanic and induced seismicity. However, many studies resulting in particularly high b-values have used relatively small datasets – both in sample size and bandwidth (magnitude range). This leads to incomplete catalogues which easily introduces a sample bias in any of the data above the magnitude of completeness (Mc), notably in the estimation of the b-value. In particular, it can be challenging to distinguish between regions of unlimited scale-free behaviour and physical roll-off at larger magnitudes. The latter model is often referred to as the modified Gutenberg-Richter (MGR) distribution.

In such datasets, an uncritical application of a maximum likelihood method to estimate the b-value could violate the implicit assumption that the data is GR distributed. If the data were not GR distributed, the methods would return a biased b-value rather than indicating that the method was inappropriate. Here we use a synthetic event catalogue of 50,000 events randomly sampled from a parent MGR distribution and fit both the GR and MGR models to the data while regularly increasing an arbitrary value of Mc. This decreases the number of events in the sampled sub-catalogue and hence the magnitude bandwidth. We apply an information criterion to determine at what point we stop being able to resolve the roll-off in the MGR distribution. For small catalogues, the information criterion (incorrectly) selects the GR model as the best fit, with a biased value of b.

We use this analysis to critique the magnitude bandwidth required to accurately distinguish a truly high b-value as opposed to a GR model being erroneously applied to rolling-off magnitudes. This shows that to correctly fit MGR to the parent MGR distribution, we require a minimum bandwidth of two orders of magnitude and a minimum sample size of 1000 events.