



## Complex yield criteria for three-dimensional granular flows

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The incompressible non-Newtonian Navier-Stokes equations have become a popular method for the simulation of granular flows [1, 2, 3]. The success of the  $\mu(I)$ -rheology [4] is certainly not insignificant for this. Within the framework of plasticity [5], this rheology can be described in terms of a yield surface and a flow rule. The yield surface is of Drucker-Prager type, parametrized with the inertial number  $I$  and the flow rule follows from incompressibility as von-Mises type.

However, the yield-strength of granular material depends on the loading type, e.g. plane strain shearing, triaxial extension or triaxial compression, which cannot be modelled with a Drucker-Prager yield criterion. Thus the Drucker-Prager yield criterion is no longer used in soil mechanics. The Mohr-Coulomb criterion or the Matsuoka-Nakai [6] criterion yield much more realistic results for the shearing failure of granular material. The difference to the Drucker-Prager criterion can be remarkable, especially if the loading type changes in time or space.

We demonstrate that the Drucker-Prager criterion within the  $\mu(I)$ -rheology can be easily replaced with an arbitrary yield surface, using the usual assumption of alignment between stretching and deviatoric stress. We introduce the Matsuoka-Nakai yield criterion and thus obtain a rheology for granular flows that appropriately predicts different yield-strengths for different loading types. This rheology is, similar to the classic  $\mu(I)$ -rheology, fully compatible with the incompressible Navier-Stokes Equations and can be implemented into CFD frameworks such as OpenFOAM [7].

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