



## **Boolean Delay Equations (BDEs) on Networks: Dynamics and Statistics**

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Boolean Delay Equations (BDEs) are semi-discrete dynamical models with Boolean-valued variables that evolve in continuous time. BDEs, unlike cellular automata, admit distinct delays between pairs of variables, and thus support more complex types of behavior; they can be classified into conservative or dissipative, in a manner that parallels the classification of ordinary or partial differential equations. Solutions to certain conservative BDEs exhibit growth of complexity in time; such BDEs can be seen therefore as metaphors for biological evolution or human history. Dissipative BDEs are structurally stable and exhibit multiple equilibria and limit cycles, as well as fractal solution sets, such as Devil's staircases and "fractal sunbursts."

BDE systems have been used as highly idealized models of natural climate variability on several time scales, as well as in earthquake modeling and prediction, and in genetics. We present recent BDE results concerning damage propagation in networks, with an emphasis on production-and-supply chain models. This setting turns out to be well adapted to investigate propagation of damages caused by a climatic or other natural disaster. The topology of the network, as well as the heterogeneity of the delays, affects the dynamics and the statistics of the system's behavior.

Complex network topologies are considered that reproduce realistic economic features, including a network of networks. Heterogeneity in the delays can cause perturbations that would otherwise vanish to amplify and lead to permanent disruptions. Difference in delays between two interacting, and otherwise resilient, networks can lead to loss of synchronization in damage propagation and thus prevent recovery. Applications to the recent network modeling of climate variability are also discussed.