



## **Diffusion of inertia-gravity waves by geostrophic turbulence**

Hossein Kafiabad, Miles Savva, and Jacques Vanneste

University of Edinburgh, Maxwell Institute for Mathematical Sciences, Mathematics, United Kingdom (kafiabad@gmail.com)

Inertia-gravity waves (IGWs) are ubiquitous in the ocean and the atmosphere. Once generated (by tides, topography, convection and other processes), they propagate and scatter in the large-scale, geostrophically-balanced background flow. In this study, we develop a theory that quantifies this scattering, and we verify its predictions through direct numerical simulations of the non-hydrostatic Boussinesq equations. The most effective scattering results from resonant interactions between two equal-frequency IGW modes and a flow (vortical, zero-frequency) mode. Since the wavevectors of equal-frequency IGWs make the same angle with the vertical, wave energy mostly remains on a cone defined by this angle in the three-dimensional wavevector space. Assuming low wave amplitudes, weak flow (small Rossby number) and a separation of spatial scales between waves and flow, we employ a WKB approach to derive a diffusion equation that describes the evolution of wave energy on this cone. The predictions of this diffusion equation agree remarkably well with simulations of the non-hydrostatic Boussinesq equations. The theory provides a time scale for the diffusion of waves and, in the stationary limit, predicts the scaling  $k^{-2}$  for the IGW energy spectrum. This scaling is consistent with atmospheric and oceanic observations in the range of scales where the preceding assumptions can be expected to hold. In the atmospheric case, this supports the interpretation recently put forward by Callies, Bühler and Ferrari of the shallow part of the Nastrom—Gage spectrum as associated with almost linear IGWs.