



The Fractional Energy Balance Equation and climate predictions, projections and Last Millenium simulations

Shaun Lovejoy and Roman Procyk
McGill University

When we consider scales much longer than the deterministic predictability limit (about 10 days), the atmosphere appears to be highly complex and thus is modelled by GCMs which are based on the nonlinear Navier-Stokes equations. Beyond their deterministic predictability limits, the GCMs effectively become stochastic and - due to the relative smallness of the forcings their responses to the forcings are essentially linear. Thus linear stochastic models may therefore be valid above the weather-macroweather transition scale of about 10 days. Eventually, at centennial or millennial or longer time scales, radiation albedo feedbacks or other mechanisms may again lead to nonlinearities. However, linear stochastic models have been used successfully for both monthly and seasonal forecasting as well as projecting temperatures to 2100. When compared to GCM based approaches, they have the advantage of being based on the real world rather than model climate.

If the dynamics are stochastic and linear, then it is important to develop the corresponding models and to put them on a strong physical basis. To date, stochastic macroweather models have been phenomenological with the more successful ones mentioned above, being based on the physical principle of scale invariance.

The conventional energy balance equation for the globally averaged temperature is a linear differential equation of first order implying exponential relaxation. It turns out that the relaxation process is rather a qualitatively different long memory, power law process with only a small fraction of the return to equilibrium occurring within the analogous (power law) relaxation time due to the hierarchy of ocean gyres and eddies each transferring heat at a rate that depends on their size and depth. Heat transfer over land also occurs in a hierarchy manner. Power laws are obtained via a seemingly trivial change in the EBE; making the differential (storage) term of fractional order: the Fractional EBE (FEBE). However, this introduces a long memory so that mathematically the problem becomes a past value problem (rather than an initial value problem) where all the past forcings contribute to the present response.

By solving the FEBE we show that the fractional relaxation time represents a transition between two power laws; between two regimes with strong but different memories characterized by theoretically related exponents: $HE = -(1/2 + HI)$. The (internal) exponent $HI \approx -0.1$, allows one to make accurate forecasts from weeks to years (macroweather), whereas the (external) exponent $HE = -(1/2 + HI) \approx 0.4$, leads to accurate decadal, centennial climate projections. The former was evaluated using hindcasts, the latter by comparisons with CMIP5 GCM integrations and on hindprojections that include the pause/slowdown/hiatus in the warming that we accurately hindproject a century earlier. Due to the success in projecting global temperatures in the future based on IPCC forcings, we use the FEBE to hindproject Last Millenium temperatures using reconstructed solar and volcanic forcings.

This linear stochastic FEBE thus unifies the internal and externally forced variability of the climate and increase understanding of the fundamental physics explaining our climate: past, present and future.