



## Upscaling in vertically fractured media

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Effective medium theory defined for multi-layered structures is very important for various applications. The Backus (1962) averaging is the famous method to compute effective medium parameters for multi-layered medium with transversely isotropic symmetry. However, for low symmetry models, this method is not valid. Roganov and Stovas (2012) showed that the effective medium computed from the stack of layers with up to triclinic symmetry can be defined by averaging of the elements of the system matrix for individual layers. In this paper, we illustrate this approach by considering a periodically layered medium consisting of layers with orthorhombic symmetry and same layers with azimuthally rotated symmetry frame. Physically, this model corresponds to the layered medium with vertical system of fractures oriented differently in different layers.

The system matrix  $M$  entering the equation of motion is defined as (Stroh, 1962)

$$M = - \begin{pmatrix} A & C_{33}^{-1} \\ B & A^T \end{pmatrix}, \quad A = C_{33}^{-1} (p_1 C_{31} + p_2 C_{32}), \quad B = \sum_{m,n=1,2} p_m p_n (C_{m3} C_{33}^{-1} C_{3n} - C_{mn}) + \rho I, \quad (1)$$

where  $C_{mn} [p, q] = c_{mp, nq}$  is the matrix of stiffness coefficients,  $\rho$  is the density, and  $I$  is 3x3 unit matrix. The azimuthally rotated stiffness coefficients matrix is given by the Bond transform,  $C_\phi = R C R^T$ , where  $R = R(\phi)$  is the rotation matrix. Considering the periodically layered medium consisting layers with orthorhombic symmetry defined by stiffness coefficients matrix  $C$  and same medium with azimuthally rotated symmetry planes,  $C_\phi$ , I obtain the effective medium with monoclinic symmetry. The effective medium parameters are controlled by the volume fraction for “rotated” medium and azimuth angle of rotation. The contour plots for effective monoclinic parameters  $\xi_j$ ,  $j = 1, 3$  responsible for rotation of S1, S2 and P wave NMO ellipses are shown in the Figure. One can see that  $\xi_1$  and  $\xi_2$  tend to zero at certain azimuth angles (different for each parameter), and parameter  $\xi_3 \leq 0$  is almost symmetric with respect to azimuth of about  $\pi/4$ . In case of isotropic or transversely isotropic symmetry of original model, the effective model reduces to original symmetry, and, therefore, the effective monoclinic parameters control the fracture distribution.

### References

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