



## **Sampling hyperspheres via extreme value theory: implications for measuring attractor dimensions**

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By using the link between extreme value theory and Poincaré recurrences, we study the relationship between the local attractor dimension  $d$  at a reference point  $\zeta$  and the topological dimension  $n$  in both random and dynamical systems. We derive a theoretical expression for the expected value of  $d(n)$  for random systems with Gaussian displacements from  $\zeta$ . Through numerical experiments, we prove the robustness of our results in random and dynamical systems characterised by different distributions. Moreover, we show that  $d$  is well smaller than  $n$  even in absence of an underlying dynamics, arguably due to the curse of dimensionality introduced by the use of the Euclidean distance to measure the proximity of each point of the attractor to  $\zeta$ . The difference between the topological and the attractor dimension is determined by the moments of the distribution of the displacements. We estimate the attractor dimension of a collection of time series issued from conceptual and real dynamical systems: we find that, while considering interacting (spatially correlated) time series reduces the attractor dimension, our theoretical result constitutes a universal scaling for  $d(n)$  also in real systems, enabling us to estimate the number of degrees of freedom frozen by the dynamics.