

## **The optimal focusing of solitons and breathers of the Gardner equation with positive cubic nonlinearity**

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Conditions of optimal (synchronized) collisions of any number of solitons and breathers are studied within the framework of the integrable Gardner equation with positive cubic nonlinearity (i.e. focusing type of nonlinearity) [1], which in the limits of small and large amplitudes tends to other long-wave models, the classic and the modified Korteweg – de Vries equations. The wave amplitude in the focal point is calculated exactly. It exhibits a linear superposition of partial amplitudes of the solitons and breathers. The crucial role of the choice of proper soliton polarities and breather phases on the cumulative wave amplitude in the focal point is demonstrated. Solitons are most synchronized when they have alternating polarities. This conclusion is similar to the previously considered case of the modified Korteweg – de Vries equation [2].

The Gardner equation is a popular model for the description of internal nonlinear waves; it is most efficient in the situations when due to the specific water density and current stratifications the quadratic nonlinear coefficient is abnormally small. In particular, the Gardner equation may possess positive cubic nonlinearity in three-layer stratification. The conditions of positive cubic nonlinearity may be found in different areas of the World Ocean [3-7]. Then solitons of different polarities (i.e. solitary waves of depression or elevation) and breathers (nonlinear wave packets) may appear. The performed research predicts that the most extreme wave events may happen in the fields of solitons which have different polarities.

Due to the generality of the Gardner equation, the considered solutions are relevant to many other physical realms.

The research is supported by the RFBR grant No. 18-02-00042. The support from the Fundamental Research Programme of RAS “Nonlinear Dynamics” is also acknowledged.

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