

The poor man's magnetohydrodynamic (PMMHD) equations: a discrete dynamical system

Tommaso Alberti (1), Giuseppe Consolini (1), and Vincenzo Carbone (2)

(1) INAF-Istituto di Astrofisica e Planetologia Spaziali, Roma, Italy, (2) Università della Calabria, Dipartimento di Fisica, Rende (CS), Italy

Dynamical systems theory is useful to describe time changes occurring in many different natural systems as the weather forecasting, the motion of billiard balls, fluid turbulence, and many other examples. As far as the turbulence is considered, the concept of “poor man’s Navier-Stokes (PMNS) equation” has been firstly introduced by Frisch in 1995 as a simple discrete logistic one-dimensional map, used as a toy model for Navier-Stokes equations to describe chaotic properties of deterministic systems.

Here, a discrete dynamical system is derived, via a Fourier-Galerkin procedure, from momentum and induction equations describing plasmas in the MHD domain. The obtained six-dimensional (6-D) map, consisting of logistic and nonlinear terms, provides useful insights into plasma dynamics and reproduces different observed behaviors when bifurcation parameters are changed. The model can be viewed as the simplest way to investigate complex time behavior of velocity and magnetic fields in the fluid-like (MHD) approximation of a plasma system. The most interesting result we found is the existence of two range of frequencies when $f < f_b$ and $f > f_b$, where f_b weakly depends of parameters, with two different power spectra $E(f)$. In all cases we found a flicker noise spectrum $E(f) \sim f^{-1}$ and a Kolmogorov-like spectrum $E(f) \sim f^{-5/3}$. This behavior represents a fixed point of our equations, also currently observed in Earth’s magnetospheric and ionospheric plasmas, space plasmas and other plasma systems. Moreover, by keeping passive the magnetic field, the map is able to describe a kinematic dynamo action. Indeed, starting from a very small seed of magnetic fluctuations, we observe that magnetic field is sustained and amplified by the velocity field through a kind of dynamo action which however acts intermittently in time. Both properties (i.e., power-law behaviors and the existence of a dynamo action) make our simple model very suitable to be used, for example, in situations where synthetic turbulent fields, with realistic properties, are required as a probe for further complex investigations (i.e., subgrid-scale models for MHD and dynamo simulations). This means that discrete dynamical systems deserve consideration for the description of plasma dynamical regimes.