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Off the beaten path: path-following methods for surface wave dispersion relations and other parametrised eigenproblems.

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In recent years, it has become apparent that oceanographic models must suitably account for local wave-current interaction effects. For vertically sheared flows –which can be prevalent in coastal regions, river deltas, and ocean surface drifts– a perturbative linear order surface wave approximation can accurately model both the dispersive properties and relevant wave dynamics.

We adapt the path-following method in [1] to calculate the dispersion relation for linear order surface waves on an arbitrary vertical shear current. This usually involves repeatedly solving the eigenproblem formed by the Rayleigh equation with Dirichlet and free-surface boundary conditions to obtain suitably many $\{k_j, c(k_j)\}$ pairs. Instead, we discretise the system and consider the eigenproblem along a path in the wave vector plane parametrised by a real scalar. This permits us to differentiate with respect to the scalar parameter; after a single initial eigenproblem solve, we can numerically integrate along the (dispersion relation) curve specified by the dominant eigenpair to obtain control points using linear solves as the Runge–Kutta $F(\cdot)$ function. Thus, we exchange many eigenproblem solves for one eigenproblem solve and a small number of linear solves. Finally, dense output is obtained using a suitable interpolant. The advantage is that after the initial setup cost of the numerical integration, many query points on the curve can be obtained accurately at almost negligible computational cost.

Possible extensions of the algorithm and the generic application of the method to other physics problems are considered.

[1] - Loisel, S. and Maxwell, P. (2018), Path-Following Method to Determine the Field of Values of a Matrix with High Accuracy, *SIAM J. Matrix Anal. Appl.* 39-4 (2018), pp. 1726-1749, doi: 10.1137/17M1148608.