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An approximated analytical solution of the Washburn equation to describe a transient vertical–capillary filling

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The classical approach to the physically–based soil hydrology is commonly framed within the hypothesis that the soil–water constitutive laws are single–valued and that they describe equilibrium processes, so that any change in the tensiometer–pressure potential would immediately induce coherent changes in the soil–water content and in the soil hydraulic conductivity. Anyway, experimental results show that in many cases disequilibrium phenomena play an important role at describing soil hydrological processes. As an example big meati and macropores tend to rapidly fill, with respect to small ones, thus contributing to onset preferential flow. Addressing the capillary bundle as a soil analogue, this behaviour may be represented by means of the classical Washburn equation, which describes the process of transient capillary filling.

Therefore, aiming at contributing to better understand the importance of disequilibrium processes in macroporous soil, we preliminarily focused on the description of the transient vertical–capillary filling, as described by the Washburn equation. The Washburn equation is a second order non–linear and non–homogeneous ordinary differential equation, for which analytical solutions are currently known for simplified cases. The Washburn equation has damped–oscillating or monotonic solutions depending on the values assumed by a bifurcation parameter which arises from its adimensionalisation.

In this work we propose an approximation of the Washburn equation by mapping, with a transformation of variables, the solution of a damped oscillator. The comparison of the approximated solution with the numerical solution of the original equation, and with experimental results collected in the literature, shows good results in most of the investigated cases. Moreover, the approximated equation and the original one are characterized by the same initial behaviour, described by means of the first term of a perturbative series, and the same value of the bifurcation parameter.

This latter property allowed to enlighten the physical meaning of the bifurcation parameter, so that the proposed analogy with a damped oscillator may be considered not only a mere analytical strategy, but it also contributes to provide an intriguing interpretation of the investigated process.