



Estimating Model Error Covariances using Particle Filters

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State-space models are the framework in data assimilation to mathematically describe the hidden state of a system by combining observations with constraints from a physical model. The formulation of these models usually involves statistical parameters that do not rely on physical constants and therefore must be estimated, since they play a central role in the performance of the data assimilation method. In particular, model error and observation error covariance matrices describe the second-order statistical properties of the system and observation stochastic equations, respectively. The model error covariance matrix \mathbf{Q} is the least constrained statistical parameter since it depends on the model physics imperfections. Moreover, a misspecification of \mathbf{Q} has a strong impact on the computation of the probability density functions involved in a particle filter algorithm, leading to an unreliable and inaccurate inference.

In this work, we propose the combination of the Expectation-Maximization algorithm (EM) with an efficient particle filter to estimate the model error covariance matrix \mathbf{Q} , using a batch of observations over a time window. The proposed method encompasses two stages: the expectation step, in which a particle filter is used with the present estimate of the model error covariance to find the probability density function that maximises the likelihood, followed by a maximization step in which this expectation is maximised as function of the model error covariance. The model evidence is written in terms of the sequential marginal likelihoods and therefore the likelihood maximization requires a particle filter and a particle smoother is not needed. Since the problem is highly nonlinear an analytical solution for this maximum is not available so that we use a fixed point iteration for the maximization step. We show that this methodology converges to the true model error covariance in stochastic twin experiments using a linear model and the Lorenz-96 system, but at different rates and with different accuracies depending on the system parameters. The extension to online estimation using the Expectation-Maximization algorithm is also discussed and evaluated