



Generalized Langevin Dynamics for Modelling the Temperature of the Earth.

Nicholas Wynn Watkins (1,2,3), Sandra Chapman (3), Aleksei Chechkin (4), Ian Ford (5), Rainer Klages (6,7), David Stainforth (1,3,8)

(1) London School of Economics and Political Science, Centre for the Analysis of Time Series, London, UK (nickwatkins62@fastmail.com), (2) Open University, Milton Keynes, UK, (3) CFSA, Physics Department, University of Warwick, UK, (4) Akhiezer Institute of Theoretical Physics, Kharkiv, Ukraine, (5) Department of Physics and Astronomy, University College London, UK, (6) TU Berlin, Germany, (7) School of Mathematical Sciences, QMUL, UK, (8) London School of Economics and Political Science, Grantham Research Institute on Climate Change and the Environment, London, UK

Energy Balance Models (EBMs) have a long heritage in climate science, including their use in modelling anomalies in global mean temperature (GMT) and the impulse response problem for idealised anthropogenic perturbation. Stochastic EBMs of the Hasselmann type allow direct treatment of climate fluctuations and noise (e.g. [1]) and are highly topical because of their use in formulating a proposed emergent constraint on climate sensitivity [2]. Considerable sophistication has now been reached in the application of multivariate and univariate stochastic models in many areas of climate, including Langevin approaches which exploit the mapping between stochastic EBMs and Langevin's original form of his equation, with GMT anomaly replacing the velocity, and the other corresponding replacements also being made.

Although still contentious, the evidence for long range memory (LRM) in GMT, and the success of fractional Gaussian noise in its prediction [3] has already brought progress from the investigation of heavy-tailed (rather than exponential) impulse response models [4,5,6]. Concurrent work with multi-box EBMs [7] is showing that LRM is less "exotic" than is sometimes believed, and can be a natural consequence of the aggregation of processes with their own characteristic timescales.

Our line of enquiry is complementary in intent to the existing approaches [3-6]. It proposes an alternative, the use of the univariate Green-Kubo "Generalised Langevin Equation" (GLE) which explicitly incorporates both non-Markovian noise and response as a tool for modelling global mean temperature [8]. We show how long memory simplifies the GLE to a fractional Langevin equation (FLE) which has not only a stochastic term identical to fractional Gaussian noise, but also a dissipation term not present in [4-6] which generalises the damping constant present in the Hasselmann formalism. We describe an EBM of this FLE-type, discuss its solutions, and relate its predictions to existing models.

- [1] Padilla et al, J. Climate (2011)
- [2] Cox et al, Nature (2018); Williamson et al, arXiv (2018)
- [3] Lovejoy et al, ESDD (2015)
- [4] Rypdal, JGR (2012)
- [5] Rypdal and Rypdal, J. Climate (2014)
- [6] Rypdal et al, Climate (2018).
- [7] Fredriksen and Rypdal, J. Climate (2017)
- [8] Watkins, GRL (2013) and chapter in Franzke & O'Kane (2017)