



Shoaling of Long Waves

Martin Oen Paulsen, Zahra Khorsand, and Henrik Kalisch

Department of Mathematics, University of Bergen, Bergen, Norway (mpa011@uib.no)

Abstract The focus of this project will be on the shoaling of long waves using linear and non-linear theory. First there will be a discussion of the model in light of previous work by [4], [1] and [2]. While the second part concerns numerical solutions for simple domains trying to reproduce shoaling curves for a periodic wave profile as done by Khorsand and Kalisch in [5]. Regarding the model we are aiming to describe gravity waves propagating from the deep sea to shallow water with a sufficiently slow variation in depth. In deep water a standard linear model will be used while the shoaling process is described by balance laws associated with the Korteweg-de Vries equation as given for example in [8]:

$$\eta_t + c_0 \eta_x + \frac{3}{2} \frac{c_0}{h_0} \eta \eta_x + \frac{c_0 h_0^2}{6} \eta_{xxx} = 0.$$

Formulating conservation laws in the KdV equations in terms of the displacement of the free surface, η will include the nonlinear effects [3]. In particular conservation of frequency, energy flux and momentum flux [3]. In addition, if time permits the momentum flux given in terms of nonlinear radiation stress [3]. And we will show that having the information from linear theory as initial data will be enough to determine the excursion of the free surface, η at some point in shallow water using nonlinear theory. Similar work has been carried out by [6], [7] using a linear formulation of energy flux. In that respect its interesting to compare the two results where Khorsand and Kalisch [5] used a nonlinear formulation of energy flux.

References

- [1] A. Ali and H. Kalisch, *Energy balance for undular bores*, C. R. Mecanique **338** (2010), 67–70.
- [2] A. Ali and H. Kalisch, *Mechanical balance laws for Boussinesq models of surface water waves*, J. Nonlinear Sci. **22** (2012), 371–398.
- [3] A. Ali and H. Kalisch, *On the formulation of mass, momentum and energy conservation in the KdV equation*, Acta Appl. Math. **133** (2014), 113–131.
- [4] R.G. Dean and R.A. Dalrymple, *Water wave mechanics for engineers and scientists*, (World Scientific, Singapore, 1991)
- [5] Z. Khorsand and H. Kalisch, *On the shoaling of solitary waves in the KdV equation*, Proc. 34th Conf. Coastal Engineering, Seoul, Korea, 2014, Vol. 34, 10 pp.
- [6] I.A. Svendsen and O. Brink-Kjær, 1972, *Shoaling of cnoidal waves*, Proc. 13th Conf. Coastal Engng, Vancouver, 1972, pp. 365–383.
- [7] I.A. Svendsen and J. Buhr Hansen, J., *The wave height variation for regular waves in shoaling water*, Coastal Engineering **1** (1977), 261–284.
- [8] G.B. Whitham, *Linear and Nonlinear Waves* (Wiley, New York, 1974).