



## Shoaling of Long Waves

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**Abstract** The focus of this project will be on the shoaling of long waves using linear and non-linear theory. First there will be a discussion of the model in light of previous work by [4], [1] and [2]. While the second part concerns numerical solutions for simple domains trying to reproduce shoaling curves for a periodic wave profile as done by Khorsand and Kalisch in [5]. Regarding the model we are aiming to describe gravity waves propagating from the deep sea to shallow water with a sufficiently slow variation in depth. In deep water a standard linear model will be used while the shoaling process is described by balance laws associated with the Korteweg-de Vries equation as given for example in [8]:

$$\eta_t + c_0 \eta_x + \frac{3}{2} \frac{c_0}{h_0} \eta \eta_x + \frac{c_0 h_0^2}{6} \eta_{xxx} = 0.$$

Formulating conservation laws in the KdV equations in terms of the displacement of the free surface,  $\eta$  will include the nonlinear effects [3]. In particular conservation of frequency, energy flux and momentum flux [3]. In addition, if time permits the momentum flux given in terms of nonlinear radiation stress [3]. And we will show that having the information from linear theory as initial data will be enough to determine the excursion of the free surface,  $\eta$  at some point in shallow water using nonlinear theory. Similar work has been carried out by [6], [7] using a linear formulation of energy flux. In that respect its interesting to compare the two results where Khorsand and Kalisch [5] used a nonlinear formulation of energy flux.

## References

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