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Mass conservation in the viscous water wave problem

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Gravity water waves are slightly viscous. Because the shear stresses should disappear on the surface of the wave, this creates a small rotational (vortical) layer at the surface. We re-examine the role of vorticity induced by the fluid viscosity in the widely used Euler-like system. In particular, we address the problem of mass conservation, a basic property that should be guaranteed in any physical system. In this case it simply denotes that as the system evolves, no water should enter or leave the considered fluid volume.

Focusing on the 2D (one horizontal, one vertical) case, our analysis reveals the equivalence of three common sets of model equations. The first set [1], uses the Helmholtz decomposition to separate the velocity field in an irrotational and a rotational part, yielding a rotational kinematic boundary condition at the surface. The second [2] also uses the Helmholtz decomposition, but makes a gauge choice that cancels out such rotational contribution in the kinematic boundary condition, at the cost of having an equation for the rotational pressure in the bulk. The third [3] formally splits the problem into two domains: the irrotational bulk and the vortical boundary layer. Here we show how the three systems are related in terms of model equations. The key in their equivalence is the correspondence between additional rotational pressure on the water surface and vorticity in the boundary layer. Moreover, we disprove the idea that mass is not conserved in commonly used systems of equations with a rotational boundary layer.

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