Riemann wave spectra in the dispersionless KdV-like models

Elena Tobisch (2), Efim Pelinovsky (3,4), Tatiana Talipova (3,4), Ekaterina Didenkulova (3,4), Arcady Dyskin (1,5)

(1) University of Western Australia, School of Engineering, Dept of Civil, Environment and Mining Engineering, Crawley, Australia (arcady_m@me.com), (2) J Kepler University, Linz, Austria , (3) Institute of Applied Physics, Nizhny Novgorod, Russia, (4) Nizhny Novgorod State Technical University n.a. R.E. Alekseev, Nizhny Novgorod, Russia, (5) Beijing Research Center for Engineering Structures and New Materials,, Beijing University of Civil Engineering and Architecture (BUCEA), China

The family of the Korteweg-de Vries-like equations $u_t + (|u|^p)_x + \text{epsilon } u_{xxx} = 0$ is discussed. Here p is an arbitrary positive constant and epsilon is a small parameter. Equations with p = 2 (classical KdV equation) and p = 3 when |u| is replaced by without absolute value (modified KdV equation) are well studied due to full integrability of both equations. The "modular" KdV equation with p = 1 describes elastic waves in bimodular media. The Shamel equation with p = 3/2 describes ion-acoustic waves with resonant electrons. In the case of the stratified fluid, parameter p can be equal to 4 and higher. Here we study the fission of the sine wave in the limit of negligible dispersion and arbitrary value of p. The analytical solution in the frame of the Riemann waves in the dispersionless Korteweg–de Vries–like equations is obtained. The Fourier spectrum of the Riemann wave is analysed; in some cases it can be expressed through the Bessel functions. We demonstrate that the wave spectrum tends to the breaking asymptotic $k^{-4/3}$ for all considered cases confirming the general result obtained early (Pelinovsky D., Pelinovsky E., Kartashova E., Talipova T. and Giniyatullin A. Universal power law for the energy spectrum of breaking Riemann waves. JETP Letters, 2013, vol. 98, No. 4, 237-241).